RESEARCH PROGRAM ON THE MANAGEMENT OF
SCIENCE AND TECHNOLOGY

Impact of Government Procurements on Employment in the Aerospace Industry

Michael Spiro*

November, 1965 #134-65

RESEARCH PROGRAM ON THE MANAGEMENT OF SCIENCE AND TECHNOLOGY

Impact of Government Procurements on Employment in the Aerospace Industry

Michael Spiro*

November, 1965 #134-65

* Assistant Professor, University of Pittsburgh

The research described in this paper was supported by Grant No. NsG 235 of the National Aeronautics and Space Administration under the general direction of Professor William H. Gruber.

ABSTRACT

An investigation of some aspects of the response of the aerospace industry to changes in the level of government orders, using mathematical tools of statistical estimation and computer simulation to determine the effect that the level of orders has on changes in employment. The industry is viewed as an information feedback system in which orders affect unfilled order backlogs and these influence the employment structure of the industry.

I. Introduction.

The study is designed to investigate some aspects of the impact on the economy of government procurements from the aircraft industry. This inquiry is an extension and a refinement of the study of Ando and Brown who have measured the lags between government obligations and aircraft production and government obligations and expenditures on aircraft.[1] In the present study, employment in the aircraft industry and in industries supplying that industry is taken as a measure of the impact.

It is hoped that the results of this study will be of interest to economists engaged in planning in government and industry and to economists engaged in research. The model which is constructed in this inquiry could serve as a useful tool for planners in government and industry. The model which is a system with only one exogenous input, orders, makes it possible to project time paths of employment and government expenditures in response to alternate streams of obligations for a fairly long horizon. It could thus facilitate the design of additional policies which would alleviate stresses that may result from a rapid increase or decrease in government procurements from the aircraft industry. Economists engaged in the design of models of the national economy will find in this study descriptions of mechanisms and estimates of lags in the aircraft industry which could serve as a component in their models.

---

[1] Ando, Brown, Solow and Kareken, Lags in Fiscal and Monetary Policy in Stabilization Policies, The Commission on Money and Credit, Prentice Hall, Englewood Cliffs, New Jersey, 1963, pp.143-145.

The study proceeds in the following sequence. A model of the aircraft industry and theories of short term employment are postulated. The hypotheses are then tested against available data and their parameters are estimated. Next a model is constructed using the tested hypotheses and the estimated parameters and an employment time series is generated with the aid of computer simulation techniques. As a further test for the validity of the model the generated employment series is compared with the actual employment data. Finally, some results of the impact of government procurements from the aircraft industry on industries supplying that industry are presented and discussed.

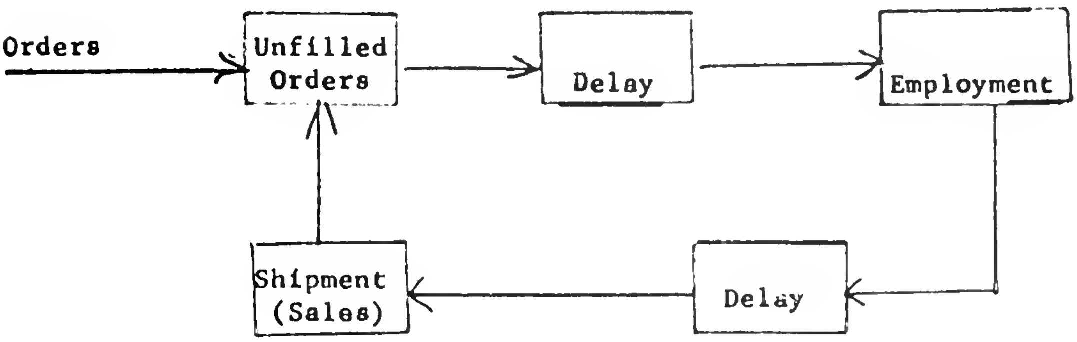
II. Models of the Aircraft Industry.

The aircraft industry is viewed as an information feedback system with the following structure. Unfilled order backlogs are the primary influence on the short-term employment decisions of the firms in the industry. The employment level determines the rates of production and shipment which in the absence of finished goods inventories corresponds to the rate of sales.[1]

---

[1] The decision to ignore finished goods inventory in this analysis is based on the following evidence: The data reveal that in 1960 finished goods inventories amounted to less than three hundred million dollars while total inventories were in excess of three billion dollars. Total shipments for the year were 13.7 billion dollars. This evidence suggests that finished goods inventories probably do not constitute a significant variable in the employment decision of the industry. It should be noted, however, that the 'in process' inventory variable which is by far the largest component of total inventories is included in the unfilled orders variable. The source for the above data is: U.S. Bureau of the Census, Annual Survey of Manufacturers: 1962, U.S. Government Printing Office, Washington, D.C., 1964, pp.357 and 47.

Two major lags are postulated to exist in this process; first, the lag between changes in unfilled orders and changes in employment, and second, the lag between changes in employment and changes in shipment. The model can best be illustrated with the aid of the following diagram:



Having described the framework that is assumed for the aircraft andustry we shall next address ourselves to the postulation of theories describing the employment decision of the industry and the relationship between employment and shipment.

II.1 Models of Short-Term Employment.

The quadratic cost function which was first suggested by Holt, Modigliani, Muth and Simon will be employed for the purpose of explaining the short-term employment decisions of the industry.[1]

[1] For a detailed exposition the reader is referred to the following source: Holt, Modigliani, Muth and Simon, Planning Production, Inventories, and Work Force, Englewood Cliffs, New Jersey, Prentice Hall, Inc., 1960.

Assume that each firm in the industry separates its investment decisions from its employment decisions and that the firm has as its objective the minimization of all costs pertaining to labor.[1] The costs of labor to be considered are: the cost of the general payroll, the costs associated with changes in the size of the workforce, and the costs of operating at over or under capacity.

If the average wage and the size of the labor force at time t are W_t and L_t respectively then the total wage bill for the period is $W_t L_t$. Let the cost of hiring and firing be proportional to the square of the change in the size of the workforce. Thus, this cost is:

$$C_2(L_t - L_{t-1})^2 \tag{1.1}$$

This type of cost function is appropriate for a firm which has monopsonistic characteristics as far as its labor market is concerned and which attaches a cost to the loss of potential capability and reputation which would result from massive layoffs.

Assume next that the firm has a desired employment level which if attained, would enable the firm to meet its contractual requirements at minimum cost. The desired employment level is defined in terms of a desired production rate, Q_t^d, and is related to it by means of a linear homogeneous production function in labor. Let L_t^d be the desired labor force and b the marginal and average productivity of labor. Then

$$L_t^d = \frac{Q_t^d}{b} \tag{1.2}$$

---

[1]The aggregation problem is discussed in Section III.

The desired production rate is determined by two primary factors; the backlog of unfilled orders at the end of the previous period, U_{t-1}, and the delivery commitments of the firm. The delivery commitments of the firm may be expressed in terms of a desired leadtime, g, which is defined as the interval of time that should elapse between the receipt of an order and the shipment of goods. The desired production rate may be approximated by dividing the backlog of unfilled orders by the desired leadtime.

$$(1.3) \qquad Q_t^d = \frac{U_{t-1}}{g} .$$

Deviations of the actual employment level from the desired level will result in costs above the minimum level because of the need for overtime, or because the labor force is not fully utilized.[1] The cost of the deviation is assumed to increase with the square of the deviation. Thus, this cost is:

$$(1.4) \qquad C_1(L_t^d - L_t)^2$$

When all these cost components are combined, and the appropriate substitutions are made, the following current cost expression is derived:

$$(2.0) \qquad C_t = C_1\left(\frac{U_{t-1}}{bg} - L_t\right)^2 + C_2(L_t - L_{t-1})^2 + W_tL_t .$$

---

[1] For a discussion of labor-hoarding the reader is referred to the following paper by Robert M. Solow: Distribution in the Long and Short Run, Conference on The Distribution of National Income, Palermo: September 1964, pp.7-15.

Assume that the firm has a short-planning horizon and chooses an employment level which will minimize current labor costs. Thus, equation (2.0) is differentiated with respect to L_t and the resulting expression is equated to zero.[1]

$$(2.1)\quad \frac{\partial C_T}{\partial L_t} = \frac{2C_1}{gb}U_{t-1} + 2C_1L_t + 2C_2L_t - 2C_2L_{t-1} + W_t = 0$$

Thus,

$$(2.2)\quad L_t = \frac{C_1}{(C_1 + C_2)bg}\ U_{t-1} + \frac{C_2}{(C_1 + C_2)}\ L_{t-1} - \frac{1}{(2C_1 + 2C_2)}\ W_t$$

and the change in the labor force is:

$$(2.3)\quad \Delta L_t = L_t - L_{t-1} = \frac{C_1}{(C_1+C_2)bg}\ U_{t-1} + \left(\frac{C_2}{C_1+C_2} - 1\right)L_{t-1} - \frac{1}{(2C_1+2C_2)}\ W_t$$

For the purpose of statistical estimation the equation is rewritten as follows:[2]

$$(2.4)\qquad \Delta L_t = a_0 + a_1U_{t-1} + a_2L_{t-1} + a_3W_t.$$

---

[1] The positive quadratic nature of the function assures the fulfillment of second order conditions.

[2] It should be noted that the only parameter which cannot be identified is either b or g.

The theory imposes some restrictions on the values that the coefficients may take. Since all the components of both a_1, a_2, and a_3 are nonnegative the model requires that a_1 will turn out to be nonnegative and a_2 and a_3 be nonpositive. Furthermore, the theory restricts the value of a_2:

$$a_2 = \frac{C_2}{C_1 + C_2} -1 = - \frac{C_1}{C_1 + C_2}$$

The expression readily suggests that a_2 must fall into the following range:

$$-1 < a_2 < 0$$

A number of additional hypotheses which are also based on the quadratic cost function concept will be tested; the detailed derivation of these hypotheses are presented in the appendix to this section. The second hypothesis assumes the previously stated cost function (2.0), but in addition contains a cost which is proportional to the square of the change in the rate of hiring or firing. The equation which is derived from this model is:

$$(3.5)\quad \Delta L_t = a_0 + a_1 L_{t-1} + a_2 L_{t-2} + a_3 W_t$$

The third hypothesis is derived from a model which assumes the cost structure stated in equation (2.0) but in which the further assumption is made that the employment decisions are based not only on the current state of affairs but also on the expectation of the firm for the foreseeable future. The resulting equation is as follows:

$$(4.10)\quad \Delta L_t = a_0 + a_1 L_{t-1} + a_2 (U_{t-1} + \sigma_t) + a_3 \sum_{i=1}^{\infty} \sigma_{t+i} z^i + a_4 \sum_{i=0}^{\infty} W_{t+i} z^i$$

O_t are current and forecasted orders and z is the weight that the firm attaches to the forecasted variables. z is a complex function of the leadtime, the rate of interest, and the various cost components.

This completes the presentation of a number of theories designed to explain the response of employment to changes in unfilled orders. In the next section a shipping function will be postulated.

II.2 The Shipping Function.

The derivation of a shipping function which in the absence of finished goods inventories will be assumed to correspond to a function describing the relationship between sales and the factors of production is based on two notions. First, a linear homogenous production function in capital and labor and second, the distributed lag concepts.[1] Assume that the firm has the following production function:

$$Q_t = bL_{t-1} + eK_{t-1} \tag{5.0}$$

Q_t is current production, L_t is the labor force and K_t is the capital stock. Let current shipment and sales, S_t, be a function of past production.

$$S_t = \sum_{i=0}^{\infty} a_i Q_{t-i} \tag{5.1}$$

Furthermore, let the weights attached to past production rates decrease geometrically.

$$a_i = a\lambda^i \qquad 0 < \lambda < 1 \tag{5.2}$$

---

[1] Koyck, Leendert, Distributed Lags and Investment Analysis, Amsterdam: North Holland Publishing Company, 1954. Solow, Robert M., "On a Family of Distributed Lags," Econometrica, Volume 28, 2 (April 1960).

Upon substitution of (5.0) and (5.2) in (5.1) the following equation is derived:

(5.3) $$S_t = \sum_{i=0}^{\infty} ab\lambda^i L_{t-i-1} + \sum_{i=0}^{\infty} ab\lambda^i K_{t-i-1}$$

and[1]

(5.4) $$S_t = b_1 L_{t-1} + e_1 K_{t-1} + \lambda S_{t-1}$$

The change in shipment is:

(5.5) $$S_t - S_{t-1} = b_1 L_{t-1} + e_1 K_{t-1} + (\lambda - 1) S_{t-1}$$

In the next sections a number of experiments will be reported which are designed to test these hypotheses and estimate their parameters.

---

[1] $b_1 = ab$ and $e_1 = ae$.

APPENDIX

In this appendix the derivation of second order lag and multi-period horizon models are presented.

Single period horizon - Second order lag model.

Assume that the firm incurs all the costs specified in the original equation (2.0), namely, wages, the cost of operating at over and under capacity, and the cost of changing the level of the labor force. However, in addition to these costs the firm also incurs a cost which is related to changes in the rate of accessions and separations. The economic justification for introducing this cost component is that the personnel and the training facilities in the firm are geared to processing a certain number of prospective and current employees and that any departure from this rate will result in costs which are proportional to the square of the deviation. This cost is represented as follows:

$$(3.1) \qquad C_3[(L_t - L_{t-1}) - (L_{t-1} - L_{t-2})]^2 = C_3(L_t - 2L_{t-1} + L_{t-2})^2$$

This cost is incorporated in the model and the following cost function is derived:

$$(3.2) \quad C_t = W_tL_t + C_1\left(\frac{U_{t-1}}{bg} - L_t\right)^2 + C_2(L_t - L_{t-1})^2 + C_3(L_t - 2L_{t-1} + L_{t-2})^2.$$

Upon differentiation with respect to L_t, setting $\frac{\partial C_T}{\partial L_t}$ equal to zero, and solving for L_t for a minimum cost employment level, the following equation is derived:

$$(3.3) \quad L_t = \frac{C_1}{(C_1 + C_2 + C_3)bg}U_{t-1} + \frac{C_2 + 2C_3}{(C_1 + C_2 + C_3)}L_{t-1} - \frac{C_3}{(C_1 + C_2 + C_3)}L_{t-2}$$

$$- \frac{1}{2(C_1 + C_2 + C_3)}W_t$$

and,

$$(3.4) \quad \Delta L_t = \frac{C_1}{(C_1 + C_2 + C_3)bg} U_{t-1} + \left(\frac{C_2 + 2C_3}{C_1 + C_2 + C_3} - 1\right) L_{t-1}$$

$$\frac{-C_3}{(C_1 + C_2 + C_3)} L_{t-2} - \frac{1}{2(C_1 + C_2 + C_3)} W_t \,.$$

For estimation purposes this equation may be represented as follows:

$$(3.5) \quad \Delta L_t = a_0 + a_1 U_{t-1} + a_2 L_{t-1} + a_3 L_{t-2} + a_4 W_t \quad .$$

The theory imposes some restrictions on the range of values that the coefficients may adopt. Since the institutional, technological, and cost parameters used are all nonnegative it can readily be seen that a_1 must be positive and a_3 must be negative. In addition, since

$$-1 < a_2 + a_3 < 0$$

amd

$$-1 < a_3 < 0$$

ot fp;;pws tjat

$$0 < a_2 < 1 \,.$$

The coefficient a_3 attains its maximum value of zero when the cost of changes in the rate of accession and separation is zero. In this case, the second order difference equation collapses into a first order difference equation.

Multiperiod Horizon Model

Assume that the firm has the cost function specified in equation (2.0). In addition, suppose that the firm has as its objective the minimization of total costs over the anticipated horizon, and that it discounts future costs by the rate of interest i. The cost function of the firm is:

$$C_T = \sum_{t=0}^{n} \left\{ \left(\frac{1}{1+i}\right)^t \left[C_1(L_t - \frac{U_{t-1}}{gb})^2 + C_2(L_t - L_{t-1})^2 + W_t L_t \right] \right.$$

However, the firm is constrained in its cost minimization by the following structural identity:

$$U_t = U_{t-1} + O_t - S_t$$

where O_t represents incoming orders and S_t represents shipments.

In the absence of finished goods inventories, all finished production is shipped. Earlier in this inquiry it was assumed that production is proportional to the work force. Now, the additional assumption will be made that shipments equal production. Thus, $S_t = bL_t$ and the constraining equation is as follows:[1]

$$U_t = U_{t-1} + O_t - bL_t$$

Next the constraint is incorporated in the model by means of the Lagrangean multiplier, λ_t . In addition the following substitution is made:

$$\frac{1}{1+i} = r .$$

The following model is now derived:

[1]This particular model assumes that there is no log between changes in employment and changes in shipment.

$$(4.0)\quad C_T = \sum_{t=0}^{n} r^t [W_t L_t + C_1(\frac{U_t}{gb} - L_t)^2 + C_2(L_t - L_{t-1})^2 + \lambda_t(U_t - U_{t-1} - O_t + bL_t)]$$

The first order conditions for minimum costs are that the partial derivatives of the endogenous variables and the Lagrangean multipliers are equated to zero.[1]

$$\frac{\partial C_T}{\partial L_t}, \frac{\partial C_T}{\partial U_t}, \frac{\partial C_T}{\partial \lambda_t} = 0 \qquad t = 1,2,\ldots n.$$

Differentiating

$$(4.1)\quad \frac{\partial C_T}{\partial L_t} = r^t W_t - \frac{2C_1}{gb} r^t U_t + 2C_1 r^t L_t + 2C_2 r^t L_t - 2C_2 r^t L_{t-1} + br^t \lambda_t$$

$$- 2C_2 r^{t+1} L_{t+1} + 2C_2 r^{t+1} L_t = 0$$

$$(4.2)\quad \frac{\partial C_T}{\partial U_t} = \frac{2C_1 r^t}{g^2 b^2} U_t - \frac{2C_1}{gb} r^t L_t + r^t \lambda_t - r^{t+1} \lambda_{t+1} = 0$$

$$(4.3)\quad \frac{\partial C_T}{\partial \lambda_t} = r^t U_t - r^t U_{t-1} - r^t O_t + br^t L_t = 0 \qquad t = 0,1,\ldots n.$$

C_T is a positive quadratic function and therefore, second order conditions for a minimum are satisfied.

---

[1]For descriptions of rigorous derivations of linear decision rules the reader is referred to any of the following sources: Holt, Modigliani, Muth and Simon, <u>Planning, Production, Inventories, and Work Force</u>, Englewood Cliffs, N.J.

(footnote cont'd., next page)

Following a division of the three sets of equations by r^t $t = 0,1,\ldots n$ we let $n \to \infty$ and apply the generating function transform

$$X(z) \equiv \sum_{t=0}^{\infty} X_t z^t .$$

z must meet the restriction $|z| < 1$

Equations (2.1), (2.2) and (2.3) are now transformed:

$$(4.4)\quad W(z) - \frac{2C_1}{gb} U(z) + 2C_1L(z) + 2C_2L(z) - 2C_2L_{-1} - 2C_2zL(z)$$

$$+b\lambda(z) - 2C_2\frac{rL(z)}{z} + \frac{2C_2rL_0}{z} + 2C_2rL(z) = 0$$

$$(4.5)\quad \frac{2C_1}{g^2b^2} U(z) - \frac{2C_1}{gb} L(z) + \lambda(z) - \frac{r\lambda(z)}{z} + \frac{r_0}{z} = 0$$

$$(4.6)\quad U(z) - U_{-1} - zU(z) -)(z) + bL(z) = 0$$

The three equations are now arranged in matrix form. The transpose of the column vector will serve as column heading.

Prentice Hall, 1960. Childs, Gerald L., Linear Decision Rules for Explaining Finished Goods Inventories and Unfilled Orders. Unpublished Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, Mass., September 1963. Holt Charles C. and Modigliani Franco, Firm Cost Structures and the Dynamic Responses of Inventories, Production, Work Force, and Orders to Sales Fluctuations and Economic Stabilization, Joint Economic Committee, Congress of the United States, 87th Congress, 1st Session, Washington, 1961, Part II, pp.51-53. Holt, Charles C. Linear Decision Rules for Economic Stabilization and Growth, Carnegie Institute of Technology, Pittsburgh, Pennsylvania, August 1960, pp.57-65.

$U(z)$	$L(z)$	$\lambda(z)$	λ_0	L_0	L_{-1}	U_{-1}	$O(z)$	$W(z)$
$-\frac{2C_1}{gb}$	$2C_1+2C_2$ $-2C_2z$ $-\frac{2C_2r}{z}+2C_2r$	b		$\frac{2C_2r}{z}$	$-2C_2$			1
$\frac{2C_1}{g^2b^2}$	$-\frac{2C_1}{gb}$	$1-\frac{r}{z}$	$\frac{r}{z}$					
$1-z$	b					-1	-1	

$= 0$

$\lambda(z)$ is now eliminated through the following row operations:

(1) Multiply row 2 by $\frac{bz}{r-z}$

(2) Add adjusted row 2 to row 1 and subtract row 2 from itself.

The following reduced matrix is derived:

$U(z)$	$L(z)$	λ_0	L_0	L_{-1}	U_{-1}	$O(z)$	$W(z)$
$-\frac{2C_1}{gb}$ $+\frac{2C_1z}{g^2b(r-z)}$	$2C_1+2C_2$ $-2C_2z$ $-\frac{2C_2r}{z}+2C_2r$ $-\frac{2C_1z}{g(r-z)}$	$\frac{br}{r-z}$	$\frac{2C_2r}{z}$	$-2C_2$			
$1-z$	b				-1	-1	

$= 0$

We now eliminate U(z) by multiplying row 2 by

$$\frac{2C_1}{gb(1-z)}\left[1-\frac{z}{g(r-z)}\right]$$ adding it to row 1 and subtracting row 2 from itself.

The resulting equation is:

$$(4.7)\quad \left\{2C_1 + 2C_2 - 2C_2z - \frac{2C_2r}{z} + 2C_2r - \frac{2C_1z}{g(r-z)} + \frac{2C_1}{g(1-z)}\left[1-\frac{z}{g(r-z)}\right]\right\}$$

$$L(z) + \left(\frac{br}{r-z}\right)\lambda_0 + \left(\frac{2C_2r}{z}\right)L_0 - 2C_2L_{-1} - \frac{2C}{gb(1-z)}\left[1-\frac{z}{g(r-z)}\right]O(z)$$

$$-\frac{2C_1}{gb(1-z)}\left[1-\frac{z}{g(r-z)}\right]U_{-1} + W(z) = 0$$

Equation (4.7) contains three unknowns: $L(z)$, λ_0 and L_0. First $L(z)$ is eliminated through a choice of z which will meet the condition that $|z| < 1$ and will also set the coefficient of $L(z)$ equal to zero.

Let

$$2C_1 + 2C_2 + 2C_2r - 2C_2z - \frac{2C_2r}{z} - \frac{2C_1}{g(r-z)}\left[1-\frac{z}{g(r-z)}\right] = 0$$

Upon the completion of routine algebraic operations the following equation is derived:

$$z^2 - \left(\frac{C_1}{C_2} + 2 + 2r + \frac{C_1}{C_2g}\right)z + \left(\frac{C_1r}{C_2} + \frac{C_1}{C_2} + 4r + r^2 + 1 + \frac{2C_1}{gC_2} + \frac{C_1}{g^2C_2}\right)$$

$$-\left(\frac{C_1}{C_2} + 2 + 2r + \frac{C_1}{C_2g}\right)\frac{r}{z} + \frac{r^2}{g^2} = 0$$

This equation has the general form

$$z^2 - Bz + C - B\frac{r}{z} + \frac{r^2}{z^2} = 0$$

Now, if it is assumed that x_1, r, $x_2 r$ are roots, it can be readily demonstrated that $\frac{1}{x_1}$ and $\frac{1}{x_2}$ are also roots of the equation.

Now $0 < r < 1$ and therefore if $x_1 < 1$ then $x_1 r < 1$ and $x_1 r$ is a root. If $x_1 > 1$ then $\frac{1}{x_1} < 1$ and $\frac{1}{x_1}$ is a root.

Thus, either $x_1 r$ or $\frac{1}{x_1}$ are roots less than unity. A similar argument can be advanced for $x_2 r$ and $\frac{1}{x_2}$. Thus, we have two roots z_1 and z_2 and each is less than unity.

The two roots are now substituted in equation (4.7) resulting in two equations, (4.8) and (4.9) and two unknowns λ_0 and L_0 and which will be solved for L_0.

First, let

$$A_1 \equiv \frac{2C_1}{gb(1-z_1)}\left[1 - \frac{z_1}{g(r-z_1)}\right] = \frac{2C_1[g(r-z_1) - z_1]}{g^2 b(1-z_1)(r-z_1)}$$

$$A_2 \equiv \frac{2C_1}{gb(1-z_2)}\left[1 - \frac{z_2}{g(r-z_2)}\right] = \frac{2C_1[g(r-z_2) - z_2]}{g^2 b(1-z_2)(r-z_2)}$$

The two equations, (4.8) and (4.9) are now arranged in matrix form:

λ_0	L_0	L_{-1}	U_{-1}	$O(z_1)$	$O(z_2)$	$W(z_1)$	$W(z_2)$
$\frac{rb}{r-z_1}$	$\frac{2C_2r}{z_1}$	$-2C_2$	$-A_1$	$-A_1$		1	
$\frac{rb}{r-z_2}$	$\frac{2C_2r}{z_2}$	$-2C_2$	$-A_2$		$-A_2$		1

$= 0$

Following the elimination of λ_0 and the appropriate algebraic operations the following equation is derived:

$$L_0 = \frac{z_1z_2}{r^2} L_{-1} + \left\{\frac{C_1z_1z_2[(1-z_2)(g(r-z_1) - (1-z_1)(g(r-z_2) - z_2]}{C_2r^2g^2b(1-z_1)(1-z_2)(z_2-z_1)}\right\} U_{-1}$$

$$+ \left\{\frac{C_1z_1z_2[g(r-z_1) - z_1]}{C_2r^2g^2b(1-z_1)(z_2-z_1)}\right\} O(z_1) - \left\{\frac{C_1z_1z_2[g(r-z_2) - z_2]}{C_2r^2g^2b(1-z_2)(z_2-z_1)}\right\} O(z-)$$

$$- \left[\frac{z_1z_2(r-z_1)}{2C_2r^2(z_2-z_1)}\right] W(z_1) + \left[\frac{z_1z_2(r-z_2)}{2C_2r^2(z_2-z_1)}\right] W(z_2)$$

Remembering that $O(z_1) \equiv \sum_{t=0}^{\infty} O_t z_1^t$ and upon the completion of this and similar substitution in the equation, and upon the introduction of appropriate definitions for the coefficients, the following equation results:

$$L_0 = \frac{z_1z_2}{r^2} L_{-1} + a_1(U_{-1} + O_0) + \sum_{t=1}^{\infty} (a_{21}z_1^t - a_{22}z_2^t)O_t - \sum_{t=0}^{\infty}(a_{31}z_1^t - a_{32}z_2^t)W_t.$$

For estimation purposes this equation is further approximated as follows:

$$L_0 = \frac{z_1z_2}{r^2} L_{-1} + a_1(U_{-1} + O_0) + a_2 \sum_{t=1}^{\infty} z^t O_t - a_3 \sum_{t=0}^{\infty} z^t W_t$$

and;

4.10) $$L_t = \frac{z_1z_2}{r^2} L_{t-1} + a_1(U_{t-1} + O_t) + a_2 \sum_{i=1}^{\infty} O_{t+i} z^i - a_3 \sum_{i=0}^{\infty} W_{t+i} z^i$$

III. Estimation of Employment Adjustment in the Aircraft Industry.

In the previous section a number of theories based on the quadratic cost function concept were advanced. This chapter presents the results of numerous experiments which were performed in order to test the theories and measure the response of employment to changes in order backlogs. The chapter opens with a discussion of some estimation problems and a description of the data. It proceeds with the presentation of the results for first and second order lag models with single period and multiperiod horizons. The concluding sections contain a description of a variety of experiments such as the testing of logarithmic models, the estimation of the response of production and non-production workers, and the estimation of the response using company rather than industry data.

III.1 Estimation problems and the data.

The theories that have been advanced contain assumptions which are stated as descriptions of the objectives and behavior of entrepreneurs, and of the cost structures of firms. The primary objective of this investigation is, however, to measure the aggregate response of employment in the industry to changes in order backlogs. The results of Grunfeld and Griliches will be used to justify our intent of using a micro model for explaining macro behavior.[1]

---

[1] Yehuda Grunfeld and Zvi Griliches, "Is Aggregation Necessarily Bad?" The Review of Economics and Statistics, Volume XLII, February 1960, pp. 1-13.

Grunfeld and Griliches argue that in situations in which the aggregation error is likely to be smaller than the specification error, and the investigator is interested in explaining aggregate behavior, he is advised to use aggregate data for estimation purposes. It is our view that these conditions hold for our model. The argument in support of this view is that our model as specified does not account for the effects on employment resulting from the interaction of firms in the industry. Specifically, the model does not contain a description of the state of the labor market at the time the firm wishes to hire. It is our expectation that the speed of hiring of any one firm will be dependent on whether other firms are hiring or laying off people.[1] On the basis of these considerations and on the basis of the practical difficulties of gathering data of companies,and the analytical difficulties of devising a composite response from individual responses, we shall proceed with the application of a micro model to macro data.

---

[1]Our attempts to support this assertion with at least one example did not prove successful. The experiment that was conducted involved the addition of aggregate industry backlogs and employment as explanatory variables for employment at Boeing Airplane Company. However, the coefficient of unfilled orders variable was insignificant. We tend to attribute this failure to a specification error. These results are discussed in some detail in Section 8 of this chapter.

The data for the regressions are derived from a variety of sources. The quarterly data on unfilled orders in the aircraft industry for the period 1948-1963 were made available by the Aerospace Industries Association and the reported source is: Census, Facts for Industry, Series M42D. The method of sampling was, however, changed in 1961. Consequently, the data from 1961 on is inconsistent with the data for prior years. The Department of Commerce could not suggest a procedure for adjusting the data and since the consistent time series covers a span of 12 years and provides 48 observations, it was decided to limit most of the study to the period 1948-1960 and not attempt to manipulate the data.

The backlog data are measured in current dollars. Since our models are formulated in real terms the question of an appropriate price index arises. The specification of a price index for the aircraft industry is a difficult problem for a number of reasons. First, the products of the industry have changed substantially during the decade of the 1950's. One can hardly regard aircraft which were constructed in the early and late parts of the decade as being the same product. In addition, the product mix of the industry has changed. While at the beginning of the decade most of the output consisted of aircraft, in later years, an increasing portion consisted of missiles and research and development. The last complication

in devising a price index arises from the nature of the market. The prime customer is the federal government and since prices on most contracts are negotiated rather than set in the market place, one can hardly speak of a price in the conventional sense.[1] The Department of Labor does not seem to publish a price index for the aircraft industry and our attempts to locate one were unsuccessful. It should be noted that Levinson, who studied price movements in numerous manufacturing sectors, indicated that a price index for the transportation industry as a group was not available and limited his discussion of prices in transportation to motor vehicles only.[2] In the absence of a price index for the aircraft industry experiments with the Department of Labor Wholesale Price Index for Electrical Machinery and with average hourly earnings of production workers in aircraft were conducted.

The data on employment and wages are taken from the Employment and Earning Statistics; the wage data are average hourly earnings of production workers.[3]

This concludes the discussion of the main sources of data; the sources of the remaining data will be cited when the regressions utilizing the data are presented.

---

[1] For a more detailed discussion of these problems the reader is referred to Peck and Sherer, The Weapons Acquisition Process: An Economic Analysis, Boston, Harvard University, 1962.

[2] Levinson, Harold M., Postwar Movement of Prices and Wages in Manufacturing Industries, Study Paper No. 21, Joint Economic Committee, Congress of the United States, 86th Congress, 2d Session, Washington, 1960.

[3] U.S. Department of Labor, Employment and Earnings Statistics, Bulletin No. 1312-1, Washington, D.C., 1963. SIC 372, pp.257,258.

III 2. First order lag, single period horizon models.

The first set of experiments is based on the first order lag, single period horizon model. In addition to the assumptions that are stated in conjunction with the specification of the model, a naive assumption about the prices of the output of the industry will be adopted. In the absence of a price index for the industry it will be assumed that prices remained constant during the 1950's. A variety of experiments which are designed to explore the implications of these assumptions for the lag structure of the model and its explanatory power are presented. The results are:[1]

$$\Delta L_t = \underset{(10.33)}{61.04} + \underset{(.002)}{.013}\, U_{t-1} - \underset{(.048)}{.325}\, L_{t-1}$$

$R^2 = .49$ $\quad S_e = 22.49$

D.W. = .83

$$\Delta L_t = \underset{(10.549)}{60.572} + \underset{(.002)}{.013}\, U_{t-1} - \underset{(.067)}{.339}\, L_{t-1} + \underset{(3.730)}{1.105}\sqrt{T}$$

$R^2 = .49$

$S_e = 22.66$

$$\Delta L_t = \underset{(18.653)}{64.646} + \underset{(.002)}{.012}\, U_{t-1} - \underset{(.065)}{.315}\, L_{t-1} - \underset{(13.822)}{3.223}\, W_t$$

$R^2 = .49$

$S_e = 22.66$

D.W. = .84

$$\Delta L_t = \underset{(46.127)}{138.367} + \underset{(.002)}{.012}\, U_{t-1} - \underset{(.065)}{.338}\, L_{t-1} - \underset{(44.521)}{77.040}\, W_t + \underset{(12.019)}{20.920}\sqrt{T}$$

$R^2 = .52$

$S_e = 22.20$ $\quad$ D.W. = .89

1 S_e is the standard error of the estimates. D.W. is the Durbin Watson Statistic.

Additional information common to all of these estimates is: $\bar{L} = 631.332$, $r(L_{t-1}, U_{t-1}) = .9579$. Furthermore, the inclusion of the trend by means of a square root of the variable consistently outperformed the results which were obtained when the variable T was included in the estimate.

A number of comments on the meaning of these results is appropriate. The explanatory power of the unadjusted unfilled orders variable is very significant. The coefficients of unfilled orders and lagged employment are very stable and are affected very little by the introduction of additional variables into the regression. Furthermore, the introduction of a trend variable alone or wages alone does not improve the fit and only the joint introduction of these two variables results in a significant coefficient for either, though neither coefficient is as large as twice the size of its standard error. The positive sign of the coefficient of $\sqrt{T}$ is puzzling and leads us to question the assumption of a constant leadtime. Our interpretation of these results is that the assumptions of fixed prices, technology, and leadtime are probably not correct but these variables interact in such a way as to compensate for each other's effect on employment and consequently make the assumptions appear correct in the first regression. An increase in productivity, *ceteris paribus*, decreases the labor force requirements given a certain backlog in comparison to what the requirements would have been without the productivity increase. The effect of an increase in the leadtime on labor requirements is similar, and so is the effect of a price increase given that the backlog is measured in current dollars. Thus, the interpretations that either prices have actually remained constant while productivity increased and the leadtime decreased, or that productivity remained constant

while prices and leadtime changed or that all three variables changed are consistent with the empirical results cited above. Thus, the only conclusion that can be drawn is that if these variables changed, they neutralized each other's effect on employment.

In the hope of gaining some insight into these questions, estimates for productivity and leadtime were plotted as a function of time. The estimate of the leadtime is presented in Chart 1 and Chart 2. The estimate of the leadtime in Chart 1 is derived by dividing unfilled orders by sales which are taken as an indication of shipments.[1] Under the strict conditions that the firm ships a fraction of its order backlog in each period, it can be demonstrated that the average leadtime is the inverse of this fraction.[2] Even if these conditions were not maintained the data strongly suggest that the leadtime declined between the beginning and the end of the decade. In Chart 2 the leadtime is estimated by plotting cummulative orders and sales. The horizontal difference of the two functions is a rough measure of the leadtime. Chart 2 also leads to the conclusion that the lead time decline during the decade. The change in the composition of the output of the industry, the shift from manufacturing and hardware goods to research and development, may account for this phenomenon. Thus, the decline in the leadtime could explain the positive sign of the coefficient of T in the last regression.

---

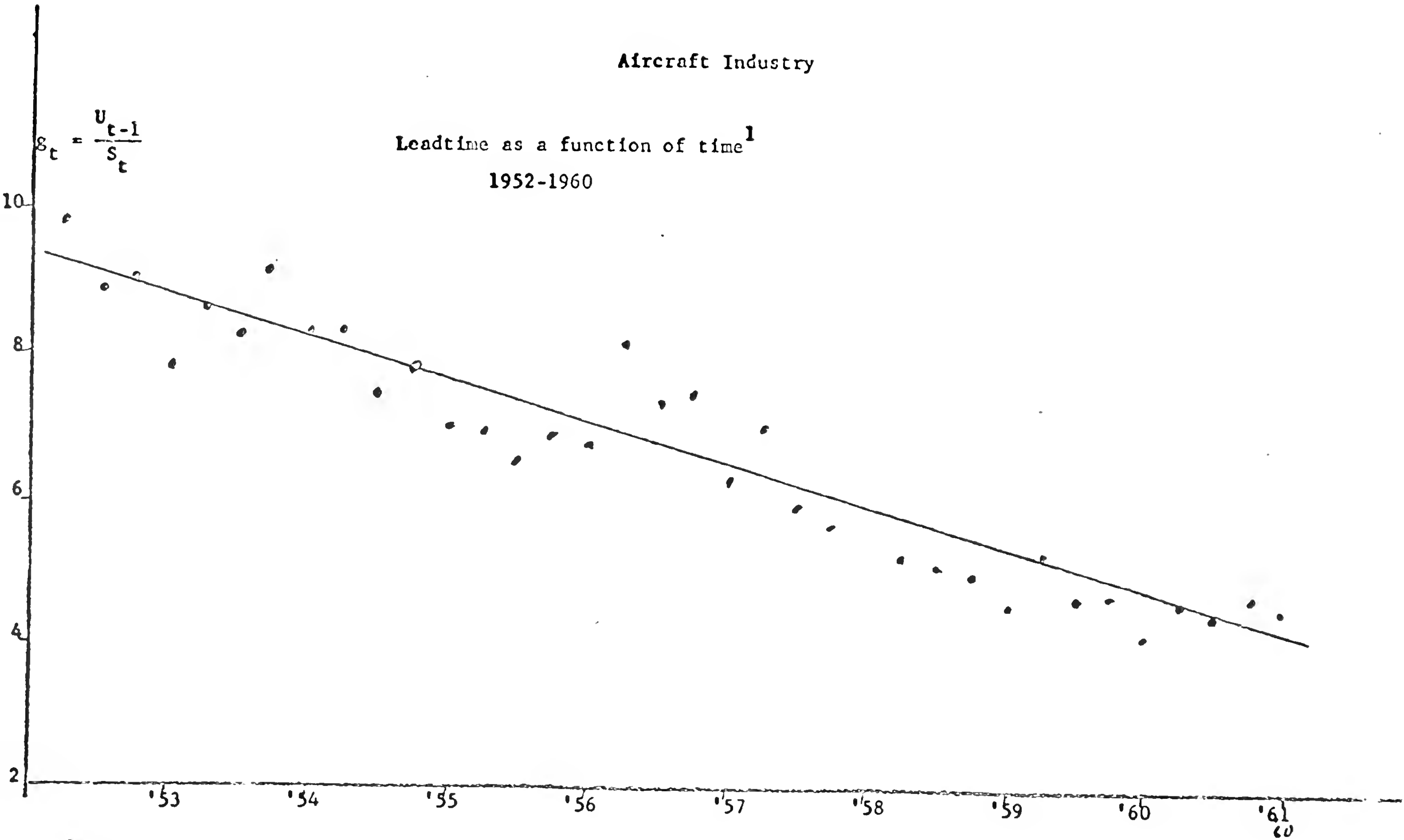
1 The assumption is made that long-run changes in actual leadtime are the result of changes in desired leadtime.

2 For a proof the reader is referred to: George Hadley, Memorandum D-28, Industrial Dynamics Research Project, School of Industrial Management, Massachusetts Institute of Technology, Cambridge, Massachusetts, January 16, 1959.

Aircraft Industry

Leadtime as a function of time[1]

1952-1960



[1] Leadtime is measured in quarters.

Chart 1

The estimate for productivity is inferred from the judgment of the economists at the Federal Reserve Board as it manifests itself in the construction of the index of industrial production for the aircraft industry. Thus, productivity estimates are derived by dividing production by employment.[1] The results are presented in Chart 3. The chart suggests that the economists in charge of the construction of the index rejected the assumption of constant productivity. Thus, the evidence that has been presented so far suggests that the leadtime declined and productivity increased during the period under study.

The next set of experiments is designed to test the implication of the relaxation of the assumption of constant prices. As a deflator of unfilled orders and wages the wholesale price index for electrical machinery is used. The results follow:

$$\Delta L_t = \underset{(11.197)}{36.334} + \underset{(.002)}{.0076}\, U_{t-1} - \underset{+(.032)}{.153}\, L_{t-1}$$

$$R = .570$$

$$S_e = 25.87$$

$$\Delta L_t = \underset{(12.012)}{34.651} + \underset{(.0025)}{.0083}\, U_{t-1} - \underset{(.069)}{.178}\, L_{t-1} + \underset{(5.626)}{2.319}\sqrt{T}$$

$$R = .572$$

$$S_e = 26.09$$

$$\Delta L_t = \underset{(67.808)}{242.148} + \underset{(.002)}{.0058}\, U_{t-1} - \underset{(.036)}{.093}\, L_{t-1} - \underset{(46.012)}{141.300}\, W_t$$

$$R = .663$$

$$S_e = 23.82$$

$$\Delta L_t = \underset{(75.994)}{402.761} + \underset{(.002)}{.0106}\, U_{t-1} - \underset{(.059)}{.268}\, L_{t-1} - \underset{(53.646)}{262.051}\, W_t + \underset{(5.986)}{21.044}\sqrt{T}$$

$$R = .748$$

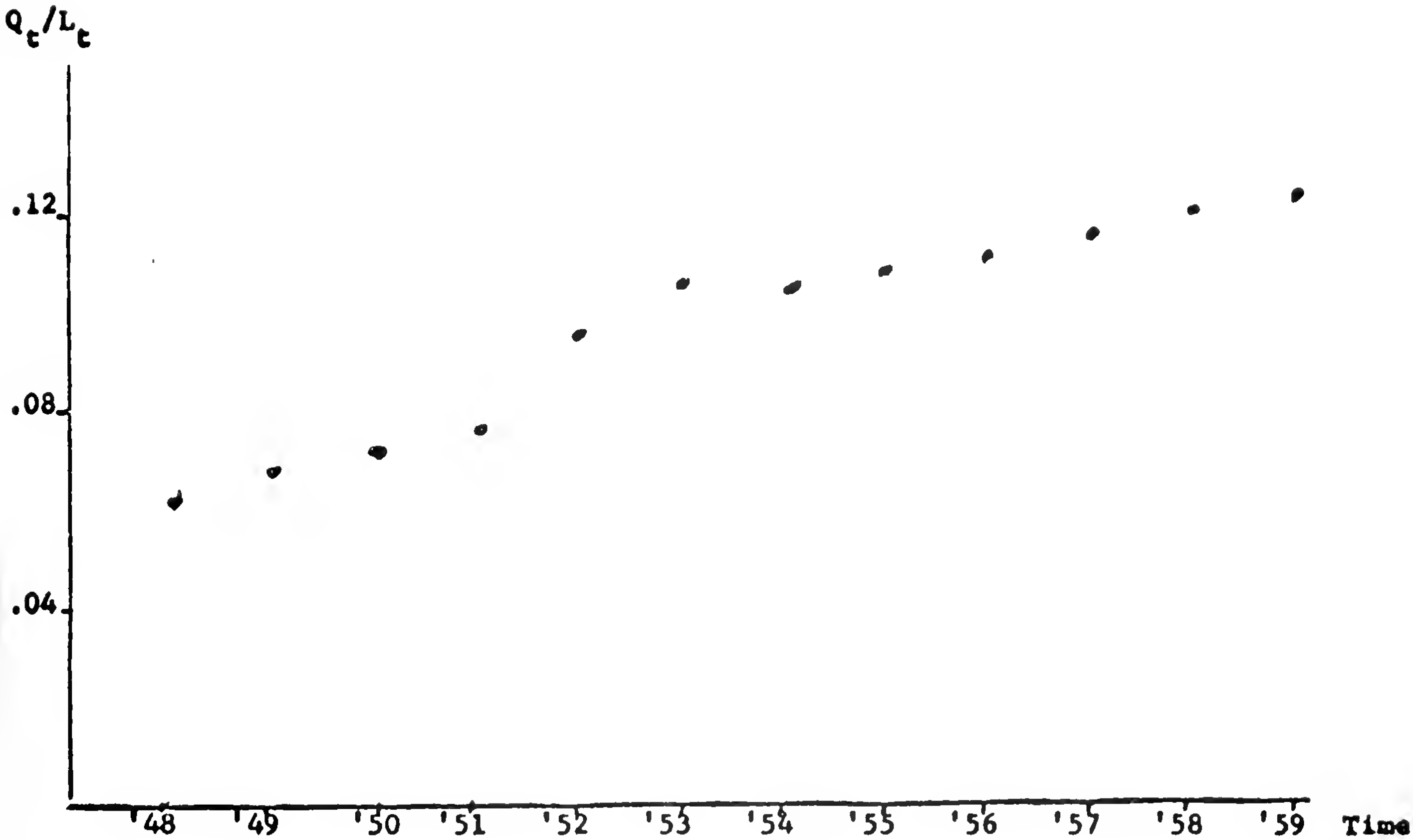
$$S = 21.33$$

---

ould be remembered, however, that manhours rather than employment sed in the construction of the index.

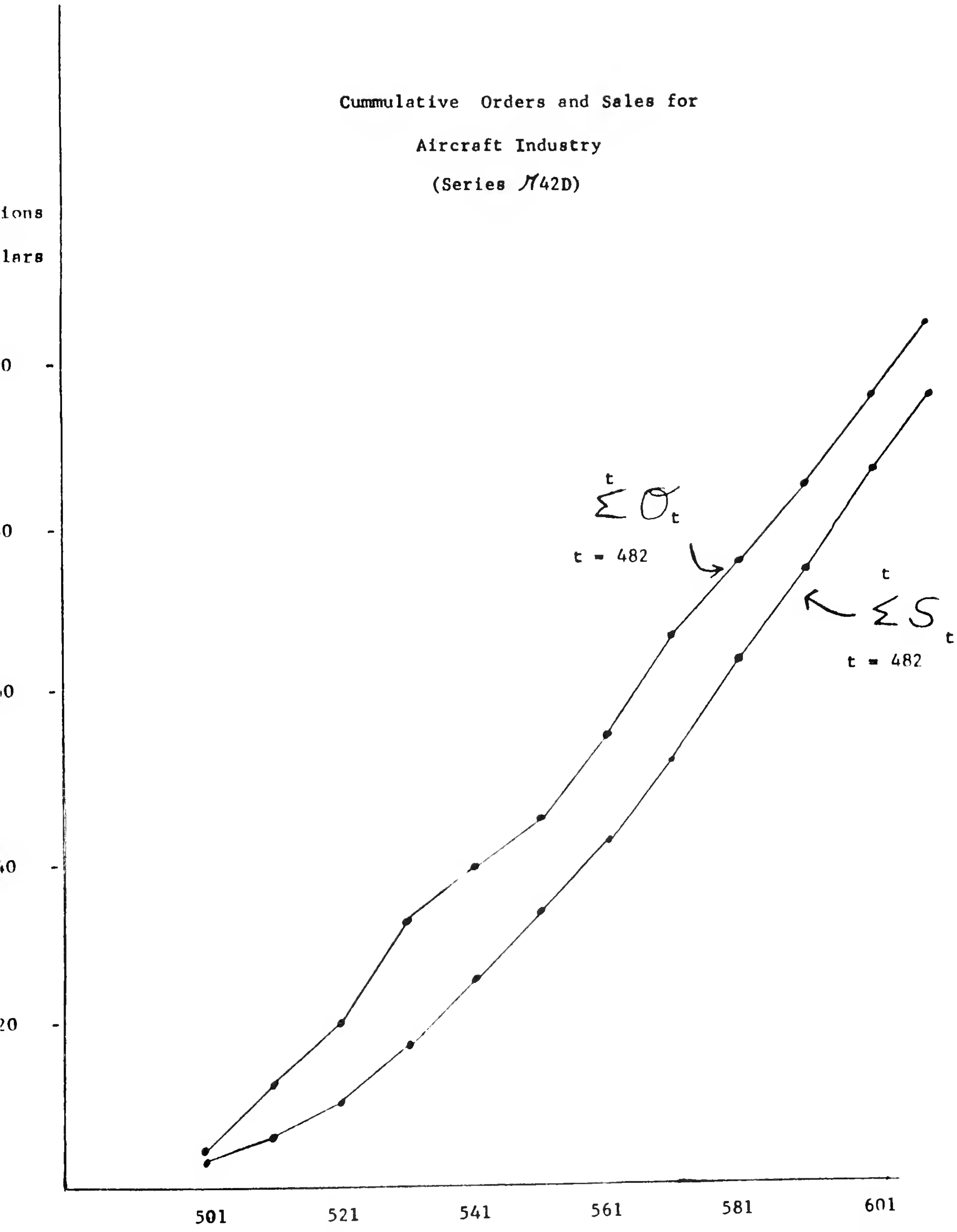
CHART 3

AVERAGE ANNUAL LABOR PRODUCTIVITY IN THE AIRCRAFT INDUSTRY AS A FUNCTION OF TIME



Q_t: Index of Industrial Production of Aircraft and Parts (SIC 372).
Source: Federal Reserve Index of Industrial Production S-86.

L_t: Total Employment in Aircraft and Parts (SIC 372).
Source: U.S. Department of Labor Employment and Earnings Statistics.


Cummulative Orders and Sales for
Aircraft Industry
(Series M42D)
ions
lars
0
0
0
0
20
$\sum_{t=482}^{t} O_t$
$\sum_{t=482}^{t} S_t$
501
521
541
561
581
601

The simple correlation between unfilled orders and employment is .8560 and is considerable lower than the value it has when unadjusted unfilled orders are used. In comparing the results of the above set of regression to the first set of results with unadjusted orders and wages, it is evident that except for the last case of the second set the fit in the first set of regressions is better than in the second set. In our judgement, the most striking result is, however, that in the last set of regressions the coefficient of L_{t-1} is quite sensitive to the presence of wages and a trend variable in the regression, and that in the last regression in which the best fit is obtained this coefficient is quite similar to the one obtained fiarly consistently in the first set of regressions when unadjusted unfilled orders are used. The values of the coefficient of L_{t-1} are .338 and -.268 and imply an average lag between the change in unfilled orders and the change in employment of 3 and 3.7 quarters respectively.

In summary to our discussion so far it may be said that though the assumptions of constant prices, technology, and leadtime may not be correct, the evidence suggests that variations in these assumed parameters have opposite effects on employment and tend to cancel out one another. The deflation of unfilled orders by the wholesale price index for electrical machi ery improves the fit slightly over what it is when unadjusted variables are used. However, the improvement is not significant enough to exclude further estimates using unadjusted data.[1]

---

[1] It should be noted that hourly earnings of production workers in the aircraft and parts industry were also used as a deflator for unfilled orders. The results, however, were inferior to the ones cited. A possible explanation is that the rise in marginal productivity of labor caused prices to rise slower than wages.

Prior to concluding the discussion of the first order-single period horizon model the response pattern implied by these models will be examined. For this purpose the first model using the unadjusted data will be used. The transient solution to this system, assuming the system starts from rest and a unit order is injected at time t_0, is:

$$L_t = .67^t$$

The percentage of total response of employment that takes place in each quarter is determined by dividing the coefficient in each period by the sum total of all coefficients which is a measure of the total effect. The total effect is:

$$\sum_{t=0}^{\infty} .67^t = \frac{1}{1 - .67} = 3.03$$

The percentage effect and the cumulative effect are presented in the following table:

Lag	0	1	2	3	4	5
Coefficient	1	.67	.45	.30	.21	.14
Per Cent of Total Response	33	23	15	10	7	5
Cumulative Response to Date (Per cent)	33	56	71	81	88	93

Thus, over 50 per cent of the total change in employment resulting from a unit input in order backlog or a shipment of one unit without a new one coming in, takes place within less than six months of the change in order backlog.

It is furthermore of interest to know not only the distribution over time of the change in employment but also the magnitude of the total change. This information is derived by solving the equation for its steady-state solution.

$$\Delta L_t = 138.367 + .012\ U - .338\ L - 77.040\ W + 20.920\ \sqrt{T} = 0$$

and

$$L = 411.6 + .036\ U - 227.9\ W + 62.0\ \sqrt{T}$$

Since employment is measured in thousands of people and unfilled orders in millions of dollars it follows from the above equation that an increase of one million dollars in the order backlog of the industry will result in an increase of thirty-six employees in the size of the labor force of the industry. The long-range elasticity of employment with respect to backlogs of unfilled orders is .68.[1] This elasticity implies increasing returns to scale.[2] When similar computations are performed using the results

---

[1] In the computations of the elasticity the mean values of the variables are used.

[2] Actually, the decline in the leadtime implies that the returns to scale are greater than what the above coefficient of elasticity suggests.

(Footnote cont'd. next page)

of the model which employs data deflated by the wholesale price index for electrical machinery an elasticity coefficient of .58 is derived. The elasticity of employment with respect to wages can also be computed in a similar way and the results are -.76 and -2.45 for the unadjusted data and the data adjusted by the price index for electrical machinery respectively. These results suggest that while considerable confidence can be placed on the explanatory power of the unfilled orders variable only very limited confidence can be placed on the explanatory power of wages. These results may be an indication of the inadequacy of our theory or of the inappropriate choice of data for the wage rate variable.

Proof: Let $E_{L,Q}$ be the elasticity of employment with respect to output and $E_{L,U}$ the elasticity of employment with respect to backlogs. Thus,

$$E_{L,Q} = \frac{\partial L/L}{\partial Q/Q}$$

and

$$E_{L,U} = \frac{\partial L/L}{\partial U/U}$$

Assume now that output is a fraction of the order backlog and that the inverse of the fraction is the average leadtime.

$$Q = \frac{U}{g}$$

$$\therefore U = gQ$$

and

$$dU = gdQ + Qdg.$$

It follows that $$\frac{dU}{U} = \frac{dQ}{Q} + \frac{dg}{g}$$

and

$$E_{L,Q} = \frac{\partial L/L}{\partial U/U - dg/g} .$$

(Footnote cont'd. next page)

III 3. Second-order lag and single-period horizon models.

The previous section reports the results of the estimation of first-order lag models. In this section the results of the estimation of second-order lag models are presented. All the experiments are designed to test the same basic logical structure and the differences among them are the consequence of different assumptions about the behavior of prices.

The result of the estimation when unadjusted data are used is:

$$L_t - L_{t-1} = \underset{(40.552)}{87.586} + \underset{(.0022)}{.0078}\, U_{t-1} + \underset{(.148)}{.215}\, L_{t-1} - \underset{(.114)}{.459}\, L_{t-2} - \underset{(37.922)}{48.875}\, W_t + \underset{(10.109)}{16{,}496}\, \sqrt{T}$$

$R^2 = .64$

$S_e = 19.33$

$D_W = 1.87$

Thus, if the leadtime is constant then $E_{L,Q} = E_{L,U}$. However, we have demonstrated earlier that the leadtime is declining and therefore $E_{L,Q} < E_{L,U}$. Hence, the observation that the coefficient $E_{L,U}$ underestimates the extent to which the returns to scale are increasing.

The result of the estimation when unfilled orders and wages are deflated by the wholesale price index for electrical machinery is:

$$L_t - L_{t-1} = \underset{(82.604)}{288.341} + \underset{(.0023)}{.0073}\, U_{t-1} + \underset{(.162)}{.148}\, L_{t-1} - \underset{(.129)}{.351}\, L_{t-2} - \underset{(57.068)}{188.149}\, W_t + \underset{(5.779)}{17.172}\sqrt{T}$$

$R_F^2 = .62$

$S_e = 19.958$

The fit in all of these cases is an improvement over the fit for the corresponding first-order lag models. Furthermore, all the signs in the equations are consistent with the theory. The transient solutions to the two equations is oscillatory but the transient practically vanishes before L_t assumes negative values in response to a positive input at U_t. The two equations imply fairly similar lag structures. The response of employment in the first system to a unit input into backlogs is:

Lag	0	1	2	3	4	5	6
Coefficient	1	1.22	1.03	.71	.41	.17	.02
Per cent of Total effect	24	29	25	17	9	6	0
Cumulative per cent of total effect[1]	24	53	78	95	104	109	109

[1]The cumulative percentage exceeds one hundred per cent because of the oscillating nature of the solution to the difference equation and the fact that the transient does not vanish completely before turning negative.

The response of the second system in which unfilled orders and wages are deflated by the wholesale price index for electrical machinery is:

Lag	0	1	2	3	4	5	6	7	8
Coefficient	1	1.15	.99	.74	.50	.32	.21	.14	.09
Per cent of total effect	18	21	18	14	8	6	4	3	3
Cumulative per cent of total effect	18	39	57	71	79	85	89	92	95

The first of these two estimates of the lag suggests that by the end of the second quarter about half of the total adjustment in the labor force will have taken place in response to a unit change in backlogs. The second estimate yields a slightly slower response and suggests that by the middle of the third quarter an adjustment of comparable magnitude will have taken place. The long-range elasticities of employment with respect to unfilled orders and wages when computed from the second-order model are about 10 per cent smaller than the corresponding values which were computed from the first-order-lag models.

It is our view that the results of the estimates of the second-order lag models support our earlier contention that the evidence does not contradict the assumption of constant prices. Thus, in the absence of a price index for the aircraft industry we shall adopt this assumption in future estimates. In the next section a model which assumes a logarithmic rather than linear lag structure will be examined.

III 4. A Logarithmic Model

The lags that have been investigated so far are all derived from linear models. An issue of interest is, however, the sensitivity of our results and inferences about the response of employment to the structure of the assumed model. In order to gain some insight into this question a logarithmic model will be postulated and tested.

Assume that the desired production rate of the firm is the unfilled order level divided by the leadtime.

$$Q_t^d = \frac{U_{t-1}}{g_t}$$

U_{t-1} is the backlog at the end of the previous quarter, g_t is the leadtime, and Q_t^d is the desired production level. The desired labor force is related to the desired production rate as follows:

$$L_t^d = \left(\frac{Q_t^d}{b_t}\right)^{\beta}$$

β is the elasticity of the desired labor force with respect to the desired production rate and b_t is an arbitrary variable. Thus

$$L_t^d = \left(\frac{U_{t-1}}{g_t b_t}\right)^{\beta}$$

Let $g_t b_t = T^{\gamma}$, and assume the following adjustment mechanism:

$$\frac{L_t}{L_{t-1}} = \left(\frac{L_t^d}{L_{t-1}}\right)^{\alpha}$$

Substituting

$$\frac{L_t}{L_{t-1}} = \left[\frac{(U_{t-1}/T^{\gamma})^{\beta}}{L_{t-1}}\right]^{\alpha}$$

For estimation purposes the equation is rewritten in logarithmic form:

$$\ln L_t - \ln L_{t-1} = a_0 + \alpha\beta \ln L_{t-1} - \alpha \ln L_{t-1} - \alpha\beta\gamma \ln T$$

When this equation is estimated using the unadjusted unfilled orders data and total employment in the aircraft industry, the results are the following:

$$\ln L_t - \ln L_{t-1} = \underset{(.119)}{.313} + \underset{(.036)}{.260} \ln U_{t-1} - \underset{(.059)}{.427} \ln L_{t-1} + \underset{(.012)}{.008} \ln T$$

$R^2 = .58$

$S_e = .0372$

$\overline{\ln L} = 6.359$

Our primary motivation for presenting this result is to point out the insignificance of the trend coefficient which is supposed to explain the variation in employment due to changes in productivity and leadtime. This result is consistent with the one derived from the estimation of the linear model using the same variables and data.

Assume now that the desired labor force is also a function of the wage rate W_t. Thus,

$$L_t^d = \left(\frac{U_{t-1}}{W^{\delta} T^{\gamma}}\right)^{\beta}$$

and

$$\frac{L_t}{L_{t-1}} = \left[\frac{U_{t-1}/W^{\delta}T^{\gamma})^{\beta}}{L_{t-1}}\right]^{\alpha}$$

Again, for estimation purposes the equation is written in logarithmic form:

$$\ell nL_t - \ell nL_{t-1} = a_0 + \alpha\beta\ell nU_{t-1} - \alpha\beta\delta\ell nW_t - \alpha\ell nL_{t-1} - \alpha\beta\gamma\ell nT$$

The results of the estimation of this model are:

$$\ell nL_t - \ell nL_{t-1} = \underset{(.116)}{.3280} + \underset{(.039)}{.233}\,\ell nU_{t-1} - \underset{(.077)}{.138}\,\ell nW_t - \underset{(.062)}{.387}\,\ell nL_{t-1} + \underset{(.019)}{.034}\,\ell nT$$

$R^2 = .61$

$S_e = .036$

$\overline{\ell nL} = 6.359$

It should be pointed out that as in the case of the linear model, the coefficient of the wage variable is insignificant in the absence of the trend variable.

The long term elasticity of employment with respect to unfilled orders and with respect to wages can readily be computed. The first elasticity is β and the second is $\delta\beta$. In the table below the elasticities which are computed from various models are presented for comparative purposes:

Steady state elasticity of employment with respect to	Linear Model[1] Unadjusted Data	Linear Model[1] Data adjusted by price index for electrical machinery	Logarithmic Model Unadjusted Data
Unfilled orders	.68	.58	.61
Wages	-.76	-2.45	-.35

These results point out again the fact that the unfilled orders variable leads to fairly stable inferences; this is not the case with the wage variable.

The study of the lag structure that is implied by the logarithmic model is more complex than the one implied by the linear model since one

[1]The wage elasticity of the linear model depends significantly on the sample observation at which it is computed. The following additional long term elasticities of employment with respect to wages were computed from the linear models:

Sample Point	Unadjusted data	Data adjusted by price index for electrical machinery
1st quarter 1950	-1.4	-6.1
1st quarter 1960	-.87	-2.5

cannot assume a unit input into unfilled orders and study the employment response while the model is in its logarithmic form. Thus, a number of approximations will have to be made.

Assume that our model is:

$$\frac{L_t}{L_{t-1}} = \left(\frac{L_t^d}{L_{t-1}}\right)^{\alpha}$$

L_t^d is the desired employment level and is a function of unfilled orders, wages, technology, and leadtime.

The steady-state solution of the model is:

$$L_s = L_s^d$$

Assume a unit increase in the desired employment level

$$L_t^d = L_s^d + \Delta L_t^d$$

$$\Delta L_t^d = \epsilon \qquad \text{when } t = 0$$

$$= 0 \qquad \text{when } t \neq 0$$

Let

$$L_t = L_s + \Delta L_t$$

ΔL_t and ΔL_{t-1} are the deviations of employment from the steady-state level at times t and t-1 respectively.

Now

$$\frac{L_s + \Delta L_t}{L_s + \Delta L_{t-1}} = \left(\frac{L_s^d + \Delta L_t^d}{L_s + \Delta L_{t-1}}\right)^{\alpha}$$

The steps that follow are based on routine algebraic operations and approximations.

$$\frac{1 + \frac{\Delta L_t}{L_s}}{1 + \frac{\Delta L_{t-1}}{L_s}} = \left(\frac{1 + \frac{\Delta L_t^d}{L_s^d}}{1 + \frac{\Delta L_{t-1}}{L_s^d}}\right)^{\alpha}$$

$$(1 + \frac{\Delta L_t}{L_s})(1 - \frac{\Delta L_{t-1}}{L_s}) = \left[(1 + \frac{\Delta L_t^d}{L_s^d})(1 - \frac{\Delta L_{t-1}}{L_s^d})\right]^{\alpha}$$

$$1 + \frac{1}{L_s}(\Delta L_t - \Delta L_{t-1}) = \left[1 + \frac{1}{L_s^d}(\Delta L_t^d - \Delta L_{t-1})\right]^{\alpha} = 1 + \frac{\alpha}{L_s^d}(\Delta L_t^d - \Delta L_{t-1})$$

Now, $L_s = L_s^d$

Therefore

$$\Delta L_t - \Delta L_{t-1} = \alpha(\Delta L_t^d - \Delta L_{t-1})$$

or

$$\Delta L_t = \alpha(\Delta L_t^d) + (1 - \alpha) \Delta L_{t-1}$$

Using this approximation, the response of employment to a unit input into desired employment which is a function of unfilled orders will be computed. The value for α which is estimated by the logarithmic model is .387.

The transient solution to the above equation is:

$$\Delta L_t = \Delta L_0 (1 - \alpha)^t$$

ΔL_0 is the deviation of employment at time t = 0.

Let $\Delta L_t^d = \frac{1}{\alpha}$, then $\Delta L_0 = 1$ and the total response is $\frac{1}{\alpha}$ or 2.58.

The response pattern of employment to a unit input into desired employment is:

Lag	0	1	2	3	4	5
Coefficient	1.0	.61	.37	.23	.14	.08
Percent of Total response	39	24	14	9	5	3
Per cent of Cumulative Total response	39	63	77	86	91	94

In comparing this response to the one that has been computed from the estimates of the linear model using identical data, one observes that this one is slightly faster than the other response. The logarithmic model suggests that 63 per cent of the total effect will have taken place by the end of the second quarter while the corresponding number for the linear model is 56 per cent. The real process is probably neither linear nor purely logarithmic and probably contains both linear and nonlinear elements. It is our view that the results of the estimate of the logarithmic model reinforce our confidence in the results derived from the linear model concerning the speed of response of employment to changes in unfilled orders in the aircraft industry.

III 5. The multiperiod horizon model.

In estimating the multiperiod model an assumption has to be made concerning the procedure that the industry uses in forecasting future orders. A review of the literature did not suggest any particular procedure. As a matter of fact, it is interesting to note that in Weston's book on market research in the defence-space industry he does not include forecasting as one of the functions of market research.[1] This exclusion is interpreted to suggest that a mechanical procedure such as a moving average would not be appropriate. For lack of a better assumption it will be supposed that with the aid of its contact men in Washington the industry

[1] Weston, J. Fred, Defence-Space Market Research, M.I.T. Press, Cambridge, Massachusetts, 1964, p.33.

has a precise notion of forthcoming orders. This assumption may not be as unreasonable as it appears since the industry might well know about forthcoming contract awards while the exact recipient of the contract may be unknown. Thus, actual orders, O_t, will be used in the regression. The results are as follows:[1]

$$L_t = -61.685 + .014(U_{t-1} + O_t) + .003O_{t+1} + .007\ O_{t+2} + .731\ L_{t-1}$$
$$(61.208) \quad (.002) \qquad\qquad (.005) \qquad (.006) \qquad (.050)$$

$R^2 = .92$

$S_e = 21.859$

The results of the estimation of the parameters of the multiperiod model are not superior to the results of the comparable single period horizon model. Furthermore, the coefficient of L_{t-1} indicates practically identical lags in both models. A number of explanations can be offered. First, the assumption made about the forecasts is incorrect. This explanation is supported by the results of a number of surveys which were conducted by the Department of Labor and which suggest considerable uncertainty in the aircraft industry about future labor requirements.[2]

---

[1]The estimation is based on undeflated data. Also, the high correlation coefficient may in part be attributed to the fact that the dependent variable is L_t and not $L_t - L_{t-1}$. The estimate is based on 34 observations of the period 1952-1961.

[2]See for instance: Aircraft and Parts Manufacturing Industry Manpower Survey No. 72, October 1955, p.4 and No. 85, February 1958, p.5.

Given this level of uncertainty and given the leadtime in the industry which is quite long in comparison to average response time of employment to changes in order backlog, one is led to question the supposition that the industry looks beyond its order backlog in determining its labor requirements. Furthermore, it can be argued that the incorporation of order backlogs rather than orders into the decision rule may be viewed as the implicit introduction of a multiperiod horizon into the model and one which best describes the behavior of the industry.

III 6. The Response of Employment to Sales

The primary motivation in measuring the response of employment to sales is to test the supposition implied in our model that sales by the private sector which correspond to expenditure by the public sector is an unsuitable variable for the measurement of the impact of procurements on the aircraft industry. The argument in support of this statement, is that the initial impact takes place prior to the expenditure phase. It is the objective of the present experiment to support this contention.

In measuring the response of employment to sales a model is employed which is similar to the linear models which have already been discussed at length. The only difference between this model and previous models is that the desired employment level is determined by sales rather than the unfilled order backlog. *Ceteris paribus*, an increase in sales results in a decrease in unfilled orders. Therefore, if our theory is correct that unfilled orders is the relevant variable for explaining employment in the

aircraft industry, then we would expect that in the absence of finished goods inventories an increase in sales will result in a decrease in employment. The results of the estimation are presented below:[1]

$$L_t - L_{t-1} = 105.02 - .036\, S_{t-1} - .022\, L_{t-1}$$
$$(42.534)\ (.009)\qquad (.054)$$

$$R^2 = .35$$

$$S_e = 24.977$$

Two aspects of these results are noteworthy. First, the sign of the coefficitent of the sales variable is consistent with our expectations. Second, a lag if about 45 quarters is implied by the coefficient of L_{t-1}. The precise meaning of this lag is not clear.[2] It appears safe to state that it is not appropriate for the measurement of the timing of the impact of government procurements on the aircraft industry.

7. The response of production and nonproduction workers to changes in order backlog.

The data for nonproduction workers are obtained by subtracting the data for production workers form total employment data in the aircraft industry. Because of results reported and discussed earlier, order backlogs at current prices are used as the explanatory variable. The results for non-production workers are:[3]

---

[1] The current estimate as well as the remaining estimates in this investigation are based on unadjusted data.

[2] It should be noted that the coefficient is highly insignificant.

[3] The high multiple correlation coefficient is in part due to the use of L_t rather than L_t as the dependent variable in the regressions in this section.

$$L_t = 6.72 + .0013\ U_{t-1} + .91\ L_{t-1}$$
$$(2.55)\ (.0003)\qquad (.019)$$

$$R = .9969$$

$$S_e = 6.74,\quad \overline{L} = 206.3$$

and

$$L_t = 4.50 + .0006\ U_{t-1} + 1.43\ L_{t-1} - .48\ L_{t-2}$$
$$(2.57)\ (.0003)\qquad (.15)\qquad (.14)$$

$$R = .9971$$

$$S_e = 6.59$$

The coefficients are significant and all fall into ranges which are consistent with the theory. The partial correlation coefficient between unfilled orders and nonproduction workers is .837 which is significantly lower than the partial between total employment and unfilled orders. The explanation is probably that in the case of total employment the leadtime change neutralizes, in part, the effect of the technological change and the resulting decline in employment per given order backlog. However, in the case of nonproduction workers this may not be the case. It is quite possible that effects generated by the technological change may have combined forces with the declining leadtime to suggest a larger change in nonproduction employment in 1960 per order backlog change than would have been expected in 1950. In any case, a more refined theory may well be in order for explaining the behavior of nonproduction workers. The response for the first order lag is:

Lag	0	1	2	3	4	5	6	7
Coefficient	1	.91	.83	.76	.69	.63	.57	.52
Per Cent Effect	9	8	7	7	6	6	5	5
Per Cent Cumulative Effect	9	17	24	31	47	43	48	53

Clearly, the response is very slow since it takes seven quarters for 50 per cent of the effect to take place.

The response derived from the second order equation is:

Lag	0	1	2	3	4	5	6	7
Coefficient	1	1.33	1.45	1.42	1.35	1.23	1.14	1.06
Per Cent of Total Effect	5	7	8	8	7	6	6	5
Cumulative Effect (Per cent)	5	12	20	28	35	41	47	52

The response is as slow as the previous one though the time pattern is different.

The steady-state solution of the above equations is:

$$L = 74.6 + .014U ,$$

which suggests that an order of one million dollars would result in the hiring of 14 nonproduction workers.

When similar experiments are performed for the <u>production workers</u> the results are as follows:

$$L_t = 54.2 + .0099\, U_{t-1} + .61\, L_{t-1}$$
$$(12.67)\quad (.0027)\qquad (.1)$$

$$R = .989$$

$$S_e = 21.63 \qquad \bar{L} = 441.35$$

and

$$L_t = 40.73 + .0072\, U_{t-1} + 1.22\, L_{t-1} - .51\, L_{t-2}$$
$$(10.94)\quad (.0023)\qquad (.15)\qquad (.11)$$

$$R = .992$$

$$S_e = 18.04$$

The statistical results of both equations are good and the magnitude of the coefficients is consistent with the theory.

The response for the first order difference equation is:

Lag	1	2	3	4
Coefficient	1	.61	.37	.22
Per Cent Effect	39	23	15	9
Per Cent Cumulative	39	62	77	86

The second order lag model yields the following response:

Lag	0	1	2	3	4
Coefficient	1	1.22	.97	.54	.17
Per Cent of Total	30	37	29	16	5
Cumulative Percentage[1]	30	67	96	112	117

[1] The percentages in excess of 100 per cent are due to the fact that the transient response is oscillatory and does not quite taper out before going into the negative range.

The steady-state solution is

$$L = 139 + .026U$$

and suggests that 26 production workers will be hired or fired with a long term increase or decrease of one million dollars in the order backlog of the aircraft industry.

The result that more production workers than nonproduction workers will be affected by a change in order backlog, and that the production worker will be affected much sooner following the change than the nonproduction workers is hardly unexpected. It is, however, interesting to note how much slower the nonproduction workers respond in comparison to the production workers. The fact that 50 per cent of the total change of production workers occurs within less than six months while it takes over twenty months for nonproduction workers to respond in a similar manner is striking.

III 8. Company versus industry analysis.

Prior to the availability of aggregate industry data this inquiry utilized data of the Boeing Airplane Company.[1] The results of the analysis using company data are different enough from the industry analysis to merit some consideration.

The results when the single period model is applied to Boeing data for the period 1950-1960 are as follows:

---

[1] The data were made available to me by Dr. Murray L. Weidenbaum, then Corporate Economist, Boeing Airplane Company, and now Professor of Economics, Washington University, St. Louis.

$$L_t = 5.77 + .0033\, U_{t-1} + .835\, L_{t-1}$$
$$(2.08)\quad (.0012)\qquad (.045)$$

$R = .9817$

$S_e = 3.599$

The response pattern that is indicated by these estimates is:

Lag	0	1	2	3	4	5	6
Coefficient	1	.835	.693	.582	.486	.406	.339
Per Cent of Total Effect	16	14	12	10	8	7	6
Per Cent Cumulative	16	30	42	52	60	67	73

An estimation of the model containing a second order lag results in the following estimates:

$$L_t = 4.8979 + .0028\, U_{t-1} + 1.1173\, L_{t-1} - .2596\, L_{t-2}$$
$$(2.02)\quad (.0012)\qquad (.1301)\qquad (.1131)$$

$R = .9839$

The response pattern is:

Lag	0	1	2	3	4	5	6	7
Coefficient	1.00	1.12	.99	.85	.69	.56	.45	.36
Per Cent of Total Effect	13	15	13	11	9	7	6	4
Per Cent Cumulative	13	28	41	52	61	68	74	78

The steady-state solution for these equations is:

$$L = 5.77 + .0033U + .835L$$

$$L = 34.7 + .02U$$

The long range elasticity is .59

When these results are compared with those derived from industry data it appears that the Boeing Company has been slower in adjusting its labor force to changes in order backlog than the industry as a whole.[1] While over four quarters elapsed before 50 per cent of the total adjustment at Boeing took place, only 2 quarters were required for the corresponding adjustment for the industry. An examination of the raw data suggests that while Boeing was slower than the industry in its response throughout the whole decade of the 50's, the major difference occurred in the speed of buildup of employment following the outbreak of the Korean War. A possible explanation for these results is that Boeing management is more conservative in the sense that it has a greater risk aversion than the industry as a whole, or that Boeing may have encountered greater difficulties in recruiting personnel than the industry because of its geographic location, and therefore responded more slowly.

Earlier in this study the view is expressed that the aggregation error in this investigation is believed to be smaller than the specification error. In hope of supporting this belief with evidence an additional

---

[1] It should be noted that Boeing data is, of course, a subset of the industry data. Mean employment for Boeing for the period is about 11 per cent of industry employment and mean Boeing backlogs are about 16 per cent of industry backlogs.

experiment was conducted. Our theory postulates that the cost of changing the level of the work force in the firm increases with the square of the change and is independent of any other variables. This assumption which is of doubtful truth on the macro level appears wrong when applied to the firm. Surely, we would expect the cost of changing the level of the work-force to be a function of the scarcity of labor in this industry if not of general business conditions. As a possible indicator for the scarcity of labor in the industry the difference is taken between aggregate desired employment, which is a function of aggregate unfilled orders and actual aggregate employment. When these variables are incorporated in the model the results of the estimation are as follows:[1]

$$L_t - L_{t-1} = \underset{(2.925)}{10.556} + \underset{(.0020)}{.0075}\, U_{t-1} - \underset{(.056)}{.140}\, L_{t-1} + \underset{(.0005)}{.0004}U^a_{t-1} - \underset{(.0165)}{.0323}L^a_{t-1}$$

$R^2 = .36$

$S_e = 3.423$

$\bar{L} = 69.17$

The insignificance of the coefficient of the aggregate unfilled orders variable may be due to a specification error. Our theory specifies that the macro variable be related in a nonlinear way to the micro variables while in the estimation a linear relationship is used. It should be noted that while the lag structure of the firm is practically unchanged from the previous estimate, the coefficient of the unfilled orders at Boeing variable

[1] U^a and L^a indicate aggregate industry variables less Boeing variables.

more than doubles with the introduction of the macro variables. Consequently, the long range elasticity of employment with respect to unfilled orders increases from .59 to 1.58. The long range elasticities of employment with respect to orders in the industry which are reported in this study have values ranging from .58 to .68. A possible conclusion is that the industry incurs increasing returns to scale while Boeing incurs decreasing returns to scale. This conclusion which is based on a limited investigation raises some interesting questions about aggregation which, however, will be left as a subject for future studies.

IV. The Response of Sales to Employment

A number of versions of the assumed relationship between changes in sales and the factors of production in the industry were tested with available data. In conjunction with these estimates a number of problems were encountered. The major difficulty is the absence of reliable information on plant and equipment in the aircraft industry. The Security and Exchange Commission has been collecting quarterly data on net plant and equipment since 1956.[1] Furthermore, some annual data of twelve leading airframe producers is also available.[2] Thus, it appeared feasible at first glance to construct a plant and equipment time series. However, further study revealed that the aircraft industry leased a large portion of its plant and equipment from the government throughout most of the decade and thus all the available 'plant and equipment' data tend to underestimate this input into production.[3] Because of these difficulties it was decided to use a time trend variable as a substitute for the capital stock variable in the regression equation. The second difficulty pertains to the deflation of the sales variable which is measured in current dollars. In the absence of a price deflator for the industry and on the basis of the conclusions which were reached earlier in this study to the effect that the evidence does not contradict the assumption of price stability it was decided to maintain this assumption in this portion of the investigation.

---

[1] Federal Trade Commission, Security and Exchange Quarterly, Financial Report of U.S. Manufacturing, Various years.

[2] Standard and Poor's Industry Survey, Aircraft, January 2, 1958, p.A18.

(Footnote 3 next page

Two versions of the sales function were estimated. A linear and a logarithmic version; first and second order lags were assumed for both versions. The results for the linear model follow:

---

[3] Stanford Research Institute estimates that in the 1957-1961 period gross property reported on balance sheets of 13 of 19 Aerospace companies representing approximately 85 per cent of sales of the industry was slightly less than government supplied property at costs. Source: Stanford Research Institute, The Industry-Government Aerospace Relationship, Volume II, Menlo Park, California, p.119.

$$\Delta S_t = -249.51 + 1.51\, L_{t-1} - .72\, S_{t-1} + 25.15\, T$$
$$\quad (85.33) \quad (.28) \qquad (.12) \qquad (5.44)$$

$R^2 = .44$

D.W. = 2.21

$$\Delta S_t = -196.19 + 1.34\, L_{t-1} - .91\, S_{t-1} + .33\, S_{t-2} + 17.68\, T$$
$$\quad (81.90) \quad (.26) \qquad (.13) \qquad (.12) \qquad (5.75)$$

$R^2 = .53$

D.W. = 1.49

The results of the logarithmic model are:

$$\ln S_t - \ln S_{t-1} = .0793 + .71\, \ln L_{t-1} - .66\, \ln S_{t-1} + .0099\, T$$
$$\quad (.24) \quad (.13) \qquad (.12) \qquad (.0027)$$

$R^2 = .48$

D.W. = 2.18

$$\ln S_t - \ln S_{t-1} = .03 + .66\, \ln L_{t-1} - .81\, \ln S_{t-1} + .0081\, T + .20\, \ln S_{t-2}$$
$$\quad (.23) \quad (.13) \qquad (.14) \qquad (.0028) \quad (.12)$$

$R^2 = .51$

D.W. = 1.65

The logarithmic model appears to have a slightly better fit than the linear model when a first order lag is assumed; however, the linear, second order lag model appears to have the best fit of the four. The first order logarithmic model enables an easy determination of the long range elasticity of shipments with respect to employment which is .71/.66 and is equal to 1.08, implying increasing returns to scale. The average response of shipment to employment can readily be computed from the first order linear model. The computed average lag is about 1.4 quarters. The trend variable which is a substitute for the capital stock variable has a positive coefficient. This result is consistent with our expectation which is based on the argument that, *ceteris paribus*, an increase in the capital stock should result in an increase in shipment. It should be noted that experiments with the use of the square root of t instead of T resulted in inferior estimates to the ones cited above.

These results could probably be improved significantly if data on capital stock were available. A more detailed breakdown of sales according to product mix, namely, missiles and aircraft, and a breakdown of employees according to occupational category would probably also have contributed to better estimates.

V. Simulation of the Aircraft Industry and Analysis of Simulation.

This inquiry has been devoted so far to the specification and estimation of models describing the behavior of short-term employment and shipment in the aircraft industry. As an additional test for the explanatory power of our

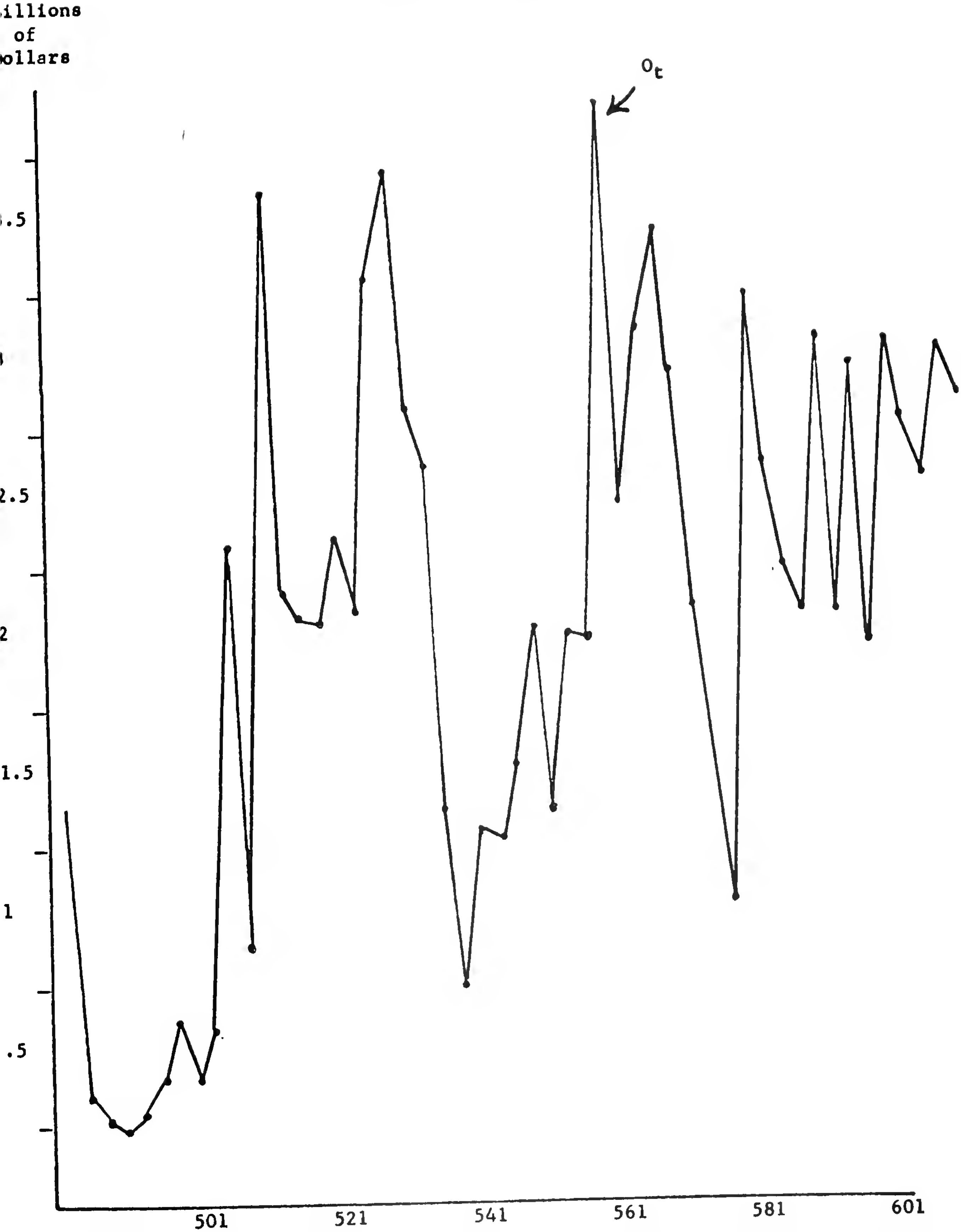
model a computer simulation experiment was conducted. A model of the aircraft industry was constructed from the specified relations and estimated parameters. Actual orders served as the only input to the model and actual values in 1948 served as initial conditions to the endogenous variables. The output of the simulation consisted of the following time series: employment, unfilled orders, and sales. It should be noted that while in the regression actual values of these variables were used as the independent variables, in the simulation the endogenously determined values were used in computing the values of these variables for the next period.[1]

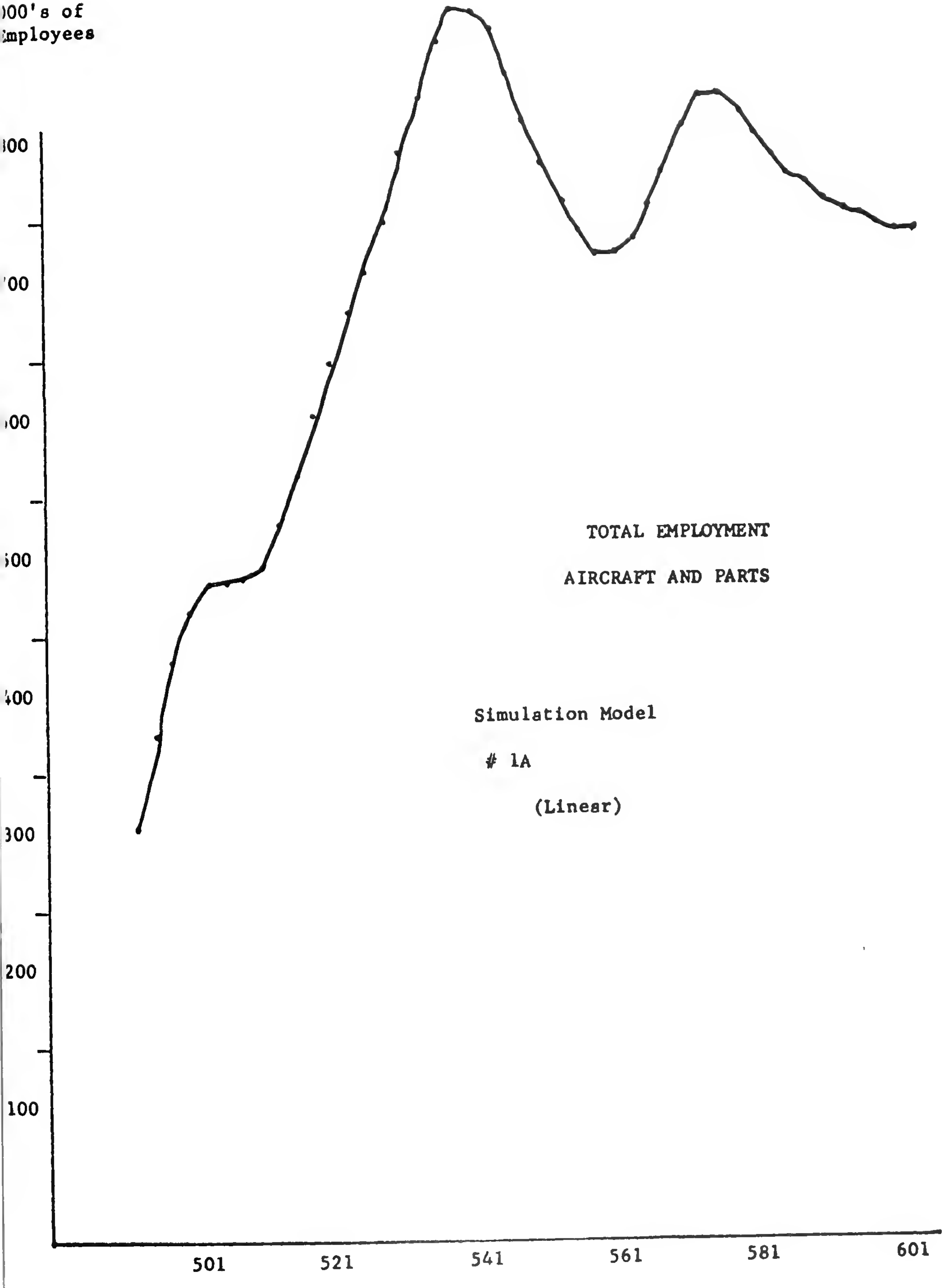
The following three charts describe the input and output of two alternative models. Chart 1 describes the actual order time series which served as the only input for the simulation. Chart 2 presents the actual employment time series. Chart 3 presents the employment time series which was generated by a linear model. Chart 4 presents the employment time series generated by a multiplicative or logarithmic model. It is our judgment that given the simplicity of these models the results are surprisingly good. As a further test for the goodness

[1]For a full discussion of this approach the reader is referred to: Cohen, Kalman J., Computer Models of the Shoe Leader, Hide Sequence, Prentice Hall, Inc., Englewood Cliffs, N. J. 1960, pp. 8-17.

ORDERS

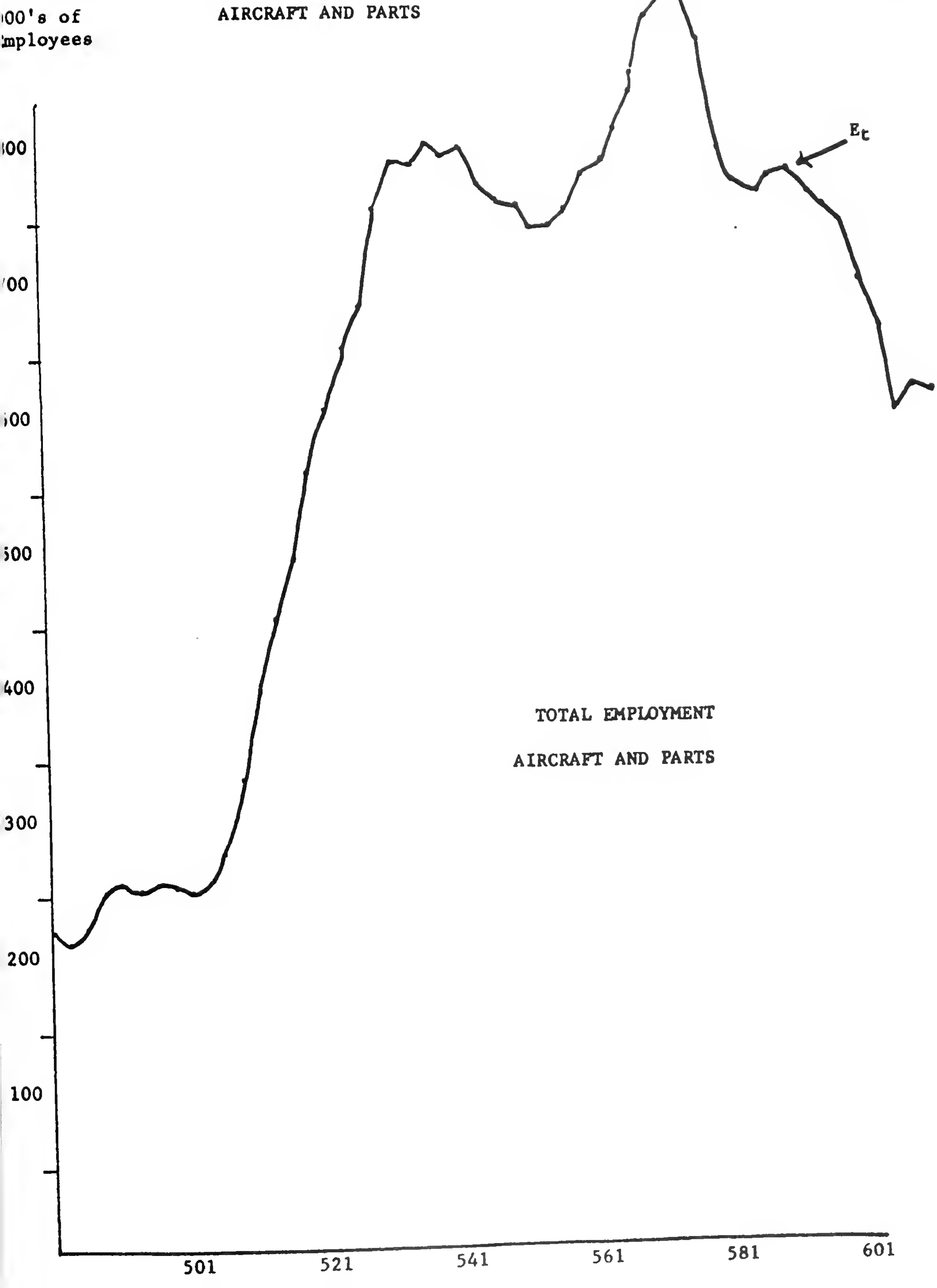
AIRCRAFT AND PARTS

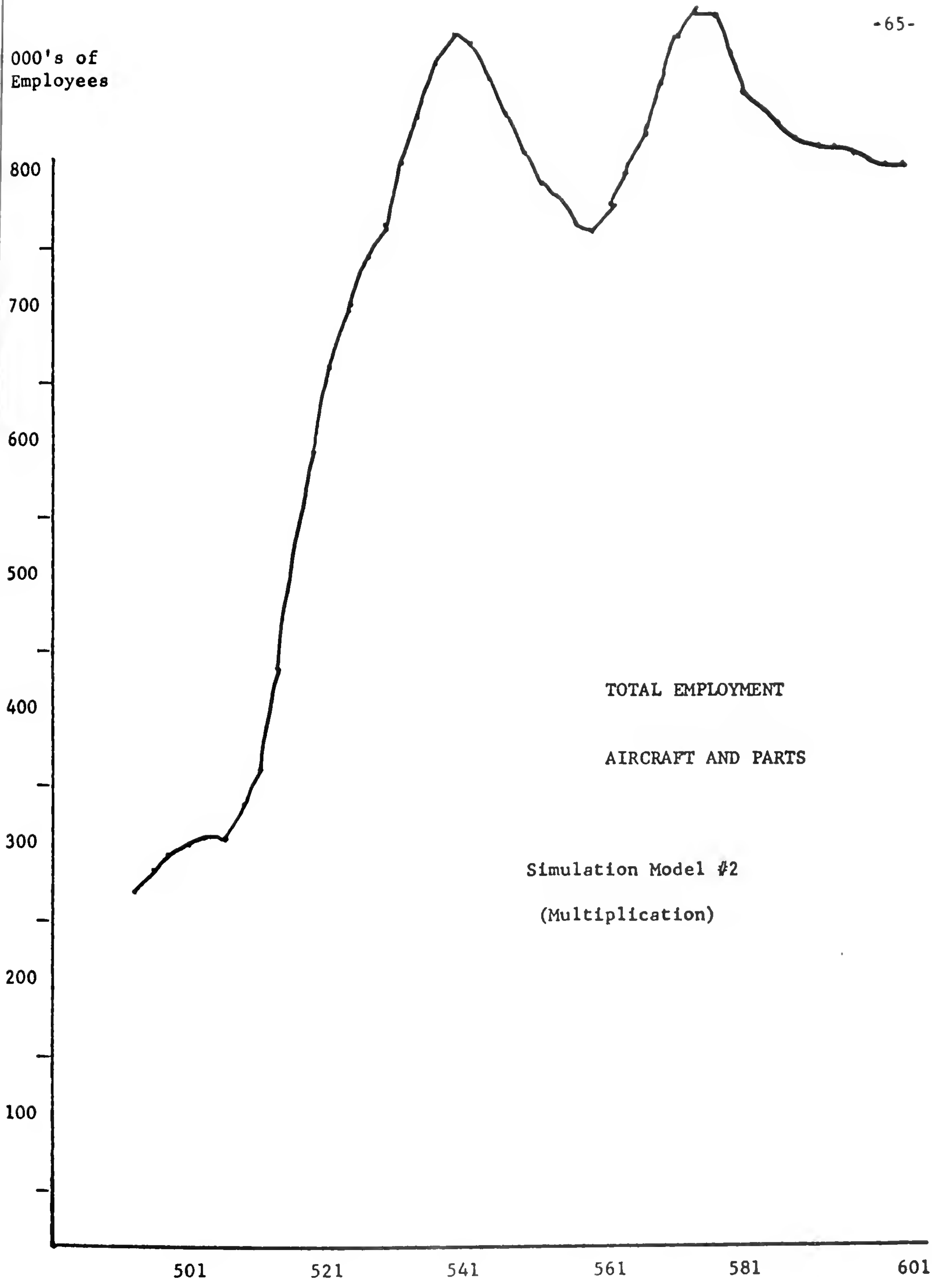



)00's of
Employees
300
700
600
500
400
300
200
100
501
521
541
561
581
601
TOTAL EMPLOYMENT
AIRCRAFT AND PARTS
Simulation Model
1A
(Linear)

TOTAL BACKLOGS--

AIRCRAFT AND PARTS




000's of
Employees
800
700
600
500
400
300
200
100
501
521
541
561
581
601
TOTAL EMPLOYMENT
AIRCRAFT AND PARTS
Simulation Model #2
(Multiplication)

of fit of these models an experiment which was suggested by Cohen and Cyert was conducted.[1] The actual employment time series was regressed against the computer generated time series. The argument for this experiment is that if the fit is good then the correlation coefficient should by high, the constant should be insignificant and the value of the coefficient of the computer generated variable should approach unity. The results of these experiments follow:

The linear model:

$$L_t^{\text{actual}} = \underset{(5.43)}{-215.15} + \underset{(.07)}{1.27}\, L_t^{\text{linear model}}$$

$R^2 = .87$

D.W. = .22

The Logarithmic Model:

$$L_t^{\text{actual}} = \underset{(25.00)}{23.91} + \underset{(.03)}{.92}\, L_t^{\text{logarithmic model}}$$

$R^2 = .94$

D.W. = .24

These results strongly suggest that the logarithmic model is superior in its explanatory power to the linear model. The high serial correlation of the residuals suggests that the deviation of the computer generated employment series and the actual employment series is not due to random fluctuations but due to some systematic errors in the model. These errors may be due to changes in productivity, leadtime and the absence of good plant and equipment data.

1 Cohen and Cyert, "Computer Models in Dynamic Economics", Quarterly Journal of Economics, Feb. '61, p. 120.

VI RESPONSE OF INDIRECT EMPLOYMENT TO CHANGES IN ORDER BACKLOG IN THE AIRCRAFT INDUSTRY

1. Introduction.

In the previous sections the first order effects on employment of government procurements were estimated. In this section a procedure will be described for measuring the second order effects. This expression is used in the sense of input-output analysis terminology except that for practical reasons we shall be going back only one step rather than trace our way through a whole maze of industries. In measuring the second order effect two primary lags have to be considered. The first is the time lag between the receipt of orders by the aircraft industry and the placement of orders for the procurements of raw materials and subassemblies with the supplying industries. The second lag is between the receipt of orders by the supplying industry and the response of employment in these industries to the new orders.

In this study the primary interest is on the secondary effect on employment and thus we are interested in the sum total of these lags and their separate values are only of incidental interest. Furthermore, practical considerations suggest the combining of these lags, since to the best of our knowledge, no data are available that would break down the orders received by an industry into their various sources. Thus, we are compelled essentially to measure the response of employment in the supplying industry to changes in order backlog in the aircraft industry.

In following this route, however, extreme caution has to be exercised and even then the results should be taken with a degree of scepticism, unless, of course, one is reasonably confident that a major portion of the output of the secondary industry is actually sold to the aircraft industry and that this basic relationship existed for the whole period under study.

Prior to the presentation of results on the indirect effect of orders placed in the aircraft industry on other industries, a discussion of the available data is required.

No reasonably satisfactory input-output model could be located. The 1947 input-output model of the United States is assumed out of date for the purpose of analysis pertaining to the aircraft industry. The 1958 input-output model will probably be of significant help but it is not yet available. In the absence of an appropriate national model an input-output model of California has been used as a guide line for identifying the major inputs into the aircraft industry and for assessing the portion of the output of the secondary industries that is supplied to the aircraft industry. The model was developed by Hansen and Tiebout.[1] This model has some shortcomings for our purpose. First, it is a regional model while we are interested in a national model. Second, it is highly aggregated. Aircraft, for that matter, falls under the category of transportation. Since, however, only .2 per cent of the output of the industry goes to final private consumption and since 61.7 percent goes

---

[1]Hansen, W. Lee and Charles M. Tiebout, An Intersectoral Flows Analysis of the California Economy. Review of Economics and Statistics, Nov. 1963, pp.409-418.

to the government, it is assumed that the predominant share of the transportation industry consists of aircraft manufacturers. The two major suppliers to the aircraft industry are ordnance and instruments, and the electrical machinery industry. Of total employment in the ordnance and instruments industry only 16.7 per cent is on transportation account, while the corresponding number for the electrical machinery industry is as low as 4.8 per cent.

Two other difficulties must be reported. While sufficient data are available on the electrical machinery industry, very little data are available on a less aggregate basis. As a matter of fact, most time series on employment in subgroups of electrical machinery are available only from 1958 on. A similar problem arises in the case of ordnance equipment. Furthermore, a problem with aircraft data must be reiterated. The size of the sample and the sampling procedure were changed in 1961 and consequently the data prior to and after 1961 are inconsistent. The differences between the two series is revealed in the respective values of backlogs for the fourth quarter of 1960 which were 12.5 and 15.3 billion dollars respectively. As a result we had practically no choice but to adjust the new series since this was the only way of getting at least a span of about five years and twenty observations for our analysis.[1]

---

[1] The adjustment consisted of the construction of a time series out of the components of the aerospace data.

VI 2. Response of Employment in ordnance to orders in the aircraft industry.

The first experiment involves a measure of the response of employment in ordnance to changes in order backlog in the aircraft industry. The results for the period 1958 through 1963 are the following:

$$L_t = \underset{(47.33)}{100.52} - \underset{(.0029)}{.0054}\, U_{t-1} + \underset{(.06)}{.858}\, L_{t-1}$$

Multiple R = .9956

The results are unsatisfactory because the sign of the coefficient of unfilled orders is negative and therefore, inconsistent with the theory. An explanation is, that while some employment in the ordnance industry is dependent on orders in the aircraft industry, the predominant share of the employees worked on projects which were substitutes for projects in the aircraft industry. Thus we have the situation of two industries predominantly competing for the defense dollar and complementing each other only to a very limited extent.[1]

The next experiment involves the use of a subset of the ordnance industry which supplies the sighting and fire control equipment for the aircraft industry (SIC 194). The results of this experiment are unsatisfactory too, insofar that the coefficient of unfilled orders in the aircraft

[1]This interpretation is also implied by Hitch who stated that the large increase in ordnance reflected both the increase in the so-called conventional weapons as well as the ordnance industry's expanding missile work. Source: Charles J. Hitch, "January 1961 Economic Report of the President and the Economic Situation and Outlook," Congress of the United States, 87th Congress, 1st Session, U.S. Government Printing Office, 1961, p.621.

industry is highly insignificant and the coefficient of the lagged variable exceeded unity in the first order lag case. An examination of the raw data suggested that employment in the sighting and fire control industry led order backlog in the aircraft industry during this period. It was furthermore observed that the aircraft and parts backlog contained an increasing portion of missiles which do not utilize fire control equipment. Thus, while the backlog of aircraft proper declined rapidly, the total decline in the backlog was dampened by an increase in missile backlog.

The final experiment with employment in the ordnance industry involves the measurement of the response of employment in fire control to backlog of aircraft and parts only, that is, exclusive of missiles. The results are as follows:

$$L_t = -.251 + .0009\, U_{t-1} + .821\, L_{t-1}$$
$$(1.97) \quad (.0004) \qquad (.11)$$

Multiple R = .989

$S_e = 1.055$

$\overline{L} = 36.6$

The coefficient of L_{t-2} is insignificant and thus the second order equation is not reported. However, the fit in the case of the first order lag is adequate and the coefficients have the correct sign and fall into the numerical range which is specified by the theory. The response pattern implied by this equation is as follows:

Lag	0	1	2	3	4	5	6
Coefficient	1	.82	.67	.55	.45	.36	.29
Per Cent of Total Effect	18	15	12	10	8	6	5
Per Cent Cumulative	18	33	45	55	63	69	74

The response pattern is significantly slower than the corresponding pattern in the aircraft industry. It is not known whether this is due to the lag which exists between the receipt of an order by the aircraft industry and the placement of an order to the fire control industry, or whether it is due simply to a slower response in the fire control industry. Attempts were unsuccessful to secure information about backlog in the sighting and fire control industry which, if available, could clarify the picture.

The steady-state solution of the equation is:

$$L = -1.38 + .0049U,$$

This equation implies that an increase of one million dollars in the backlog of aircraft proper would result in the eventual hiring of five employees in the sighting and fire control industry.

3. Response of Employment in Electrical Machinery to Orders in the Aircraft Industry.

Great difficulties were encountered in measuring the response of employment in electrical machinery to order backlog in the aircraft industry. The difficulties are due to the very small percentage of total employment in the electrical machinery industry that is dependent on aircraft procurements (even for California the estimate is as low as 4.8

per cent). Thus, one would hardly expect aircraft procurements to explain employment in electrical machinery. Furthermore, even if the statistical results are good, one should attribute a good fit not to the explanatory value of aircraft backlogs but rather to the fact that both industries happen to experience a similar growth pattern.

A number of experiments were conducted with employment data in subgroups of electrical machinery. SIC 3662, which combines radio and TV communication equipment, is assumed in a Department of Labor publication to combine the military space and commercial product categories.[1] Since no further breakdown could be located this series was used. However, a casual examination of employment in this category showed that employment over the period 1958-1963 increased while total aircraft industry backlogs decreased. Thus, it appears that a number of forces seem to be operating. It is possible that the non-military business is a significant portion of total output of this category and that its behavior differed from the military business. Even if this is true, and this could not be established, little could be done because of the composition of the data. On the other hand, it is possible that the electronic composition of the aircraft backlog may have changed over time. The Electronic Industry Association estimates that electronic components comprise 21 per cent of the dollar value of aircraft and 41 per cent of the dollar value of missiles.[2] In addition,

---

[1] Employment Outlook and Changing Occupational Structure in Electronic Manufacturing, U.S. Department of Labor Bulletin 1363, p.48.

[2] Electronic Industry Association, 1963 Yearbook, p.38.

missiles have been occupying an increasing share of the backlog of the aircraft industry. The combination of these factors could explain an increase in employment in SIC 3662 when backlogs in aerospace decreased. As a result of these considerations it was decided to measure the response of employment in this category to a category in the backlog which was called 'other products and services' through the year 1960 and 'missiles and space vehicles and parts' from that year on. The results were as follows:

$$L_t = \underset{(9.13)}{28.52} + \underset{(.0030)}{.0005}\, U_{t-1} + \underset{(.04)}{.91}\, L_{t-1}$$

Multiple R = .990

The high multiple correlation coefficient is due to the serial correlation of the employment data. Since the coefficient of the unfilled orders backlog variable is highly insignificant it can only be concluded that the data do not support this model and no estimate of the lag structure of the response of employment in SIC 3662 to changes in order backlogs in the aircraft industry can be derived.

This completes the presentation of the estimation of lags in the response of employment in industries which act as suppliers to the aircraft industry to changes in order backlogs in the aircraft industry. The results of the estimation of the indirect effect are disappointing and worthy of further investigation and analysis. It is, however, doubtful whether significantly better results can be obtaindd in view of the high level of aggregation of the employment time series.

VII Conclusions.

This inquiry was designed to study the nature of the effect of government procurements from the aircraft industry. Following the specification of an overall framework for analysis a theory for explaining the adjustment of employment is advanced. The results of experiments suggest that the theory which was based on the quadratic cost function concept explained well the behavior of employment in the aircraft industry. The estimated average lag in the response of employment to orders is nine to ten months with over fifty per cent of the response occurring within the first six months. The results also suggest that, *ceteris paribus*, a change of one million dollars in order backlogs results in the hiring or firing of thirty-five to forty employees. The estimated lag between changes in employment and changes in sales is about five months. Thus the total estimated lag between orders and sales which correspond to government obligations and expenditures is about fifteen months. This estimate is somewhat smaller than that of Ando and Brown who estimated this lag to be about twenty months.[1]

The study may, however, be of theoretical and practical interest beyond the information which it yielded on the response of employment in the aircraft industry. The synthesis of the aircraft industry which was performed using computer simulation techniques may provide additional power to our ability to test theories.[2] Furthermore, the model may prove useful to planners in government and industry.

[1] Ando and Brown, Solow and Kareken, *Lags in Fiscal and Monetary Policy in Stabilization Policies*, The Commission on Money and Credit, Prentice Hall, Englewood Cliffs, N.J., 1963, p.143.

[2] This statement is just a conjecture, as Professor Edwin Kuh has pointed out, and requires a rigorous mathematical proof.